# WHY HAS THE U.S. ECONOMY STAGNATED SINCE THE GREAT RECESSION?

Yunjong Eo and James Morley*

*Abstract*—Since the Great Recession in 2007–2009, U.S. real GDP has failed to return to its previously projected path, a phenomenon widely associated with secular stagnation. We investigate whether this stagnation was due to hysteresis effects from the Great Recession, a persistent negative output gap following the recession, or slower trend growth for other reasons. To do so, we develop a new Markov-switching time series model of output growth that accommodates two different types of recessions: those that permanently alter the level of real GDP and those with only temporary effects. We also account for structural change in trend growth. Estimates from our model suggest that the Great Recession generated a large, persistent negative output gap rather than any substantial hysteresis effects, with the economy eventually recovering to a lower trend path that appears to be due to a reduction in productivity growth that began prior to the onset of the Great Recession.

## I. Introduction

THE slow growth of the U.S. economy in the wake of the Great Recession in 2007–2009 has led to a revival of earlier notions of secular stagnation (Hansen, 1939) and hysteresis (Blanchard & Summers, 1986). There are different theories of secular stagnation, but Summers (2014, 2015) emphasizes the role of inadequate demand. According to his view, the global financial crisis in 2008 saw an unwinding of a financial bubble that had propped up the world economy. In its absence and in the face of the zero-lower-bound that prevented a further lowering of interest rates, inadequate demand caused the economy to grow at a slower rate than otherwise. This theory is related to the idea that inadequate demand resulting from the Great Recession may have produced hysteresis or even "super-hysteresis" effects (Ball, 2014; Guerron-Quintana & Jinnai, 2019) that have permanently lowered both the level and growth path of economic activity. Using data from 23 countries, Blanchard, Cerutti, and Summers (2015) document that many recessions have led to such effects, although they acknowledge that the causality could reflect supply shocks and financial crises producing both a recession and subsequent stagnation. Cerra and Saxena (2017) argue that recessions always have negative permanent effects on the level of aggregate output and question the relevance of the concept of an output gap in the first place, including in explaining weak economic activity and sluggish growth following the global financial crisis.

A contrasting view of secular stagnation, emphasized by Gordon (2015, 2016), is that it reflects supply-side forces such as slower productivity growth and demographic changes that started before the Great Recession. Notably, Fernald et al. (2017) use a growth accounting decomposition and find that, once allowing for cyclical effects, the slow growth in the U.S. economy since the Great Recession can be related to slow growth of total factor productivity and a decline in labor force participation, with both phenomena starting prior to the onset of the recession and not obviously connected to the financial crisis. Supporting this view, a few recent empirical studies have estimated a structural break in U.S. trend growth in the mid-2000s prior to the Great Recession, including Grant and Chan (2017), Antolin-Diaz, Drechsel, and Petrella (2017), and Kamber, Morley, and Wong (2018). However, an ability to reject that the slowdown actually occurred during the Great Recession, rather than before, is unclear from this literature.

In this paper, we develop a flexible new nonlinear time series model that allows us to examine the empirical support for competing views surrounding why U.S. real GDP did not return to its projected path prior to the Great Recession. In particular, we investigate whether the stagnation of the economy was due to level and growth hysteresis effects from the Great Recession, a persistent negative output gap following the recession, or slower trend growth for other reasons. Building on Hamilton (1989), Kim and Nelson (1999a), Kim, Morley, and Piger (2005), and Eo and Kim (2016), our univariate Markov-switching model of real GDP growth allows a given recession to either permanently alter the level of aggregate output (an L-shaped recession) or only have a temporary effect (a U-shaped recession).[1] We also account for structural change in trend growth. In particular, using the testing procedures from Qu and Perron (2007), we find an estimated reduction in the long-run growth rate of U.S. real GDP in 2006Q1. When allowing for this break in our Markov-switching model, we find that the Great Recession was U shaped, generating a negative and persistent output gap rather than any substantial level hysteresis effects, with the economy eventually recovering to a lower-growth trend path. However, our finding about the nature of the Great Recession is robust to the reduction in trend growth occurring earlier, allowing for more complicated

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*Eo: Korea University; Morley: University of Sydney.

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[1]The univariate approach often taken in the literature on nonlinear output growth dynamics makes the implicit oversimplifying assumption of a common propagation for all underlying symmetric shocks to aggregate output, regardless of their source. However, it has the benefit of allowing for a tightly parameterized but still sophisticated specification of dynamics for asymmetric shocks, in our case allowing for different dynamics for two types of recessions.

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patterns of structural change, or even assuming no structural change at all, although the model without a break in trend growth produces persistently downward-biased forecast errors after the Great Recession. Compared to the analysis using Qu and Perron (2007) procedures, the precision of our inference that the break occurred before the Great Recession is sharpened considerably by taking into account nonlinear dynamics with our Markov-switching model. Notably, we are able to formally reject that the slowdown in trend growth occurred after 2006Q2, and therefore does not appear to be due to the Great Recession. Furthermore, we find that the apparent timing of the slowdown is more consistent with a reduction in productivity growth than demographic factors.

Our analysis is related to Huang, Luo, and Startz (2016), who also consider a univariate time series model with two different types of recessions but determine the prevailing regime using NBER dates and assume a given recession is predetermined as being either L or U shaped. Our Markov-switching model is more directly an extension of Hamilton (1989), Kim and Nelson (1999a), and Kim, Morley, and Piger (2005) to allowing two different types of recessions by modeling regimes as being stochastic. We believe this is a more natural assumption given that the exact timing and nature of recessions are not predetermined in practice. This also leads to a different result than that of Huang, Luo, and Startz (2016) in terms of categorizing the Great Recession as being U shaped rather than L shaped. Our model is also somewhat related to Kim and Murray (2002), Kim and Piger (2002), and Kim, Piger, and Startz (2006), who consider multivariate unobserved components models with Markov switching in both the trends and cycles of panels of macroeconomic time series, thus allowing for L- and U-shaped recessions. However, those models make assumptions about the correlations between permanent and transitory movements, which implicitly place strong restrictions on the variance of the stochastic trend in aggregate output that do not appear to be supported by the data (Morley, Nelson, & Zivot, 2003).

The rest of this paper proceeds as follows. In section II, we provide background evidence for nonlinearity and structural breaks in U.S. real GDP. In section III, we present the details of our new Markov-switching model and show how it can generate both L- and U-shaped recessions. In section IV, we report estimates for a benchmark version of our model and examine implications for why real GDP has stagnated since the Great Recession. In section V, we consider alternatives to our benchmark model in order to investigate the robustness and interpretation of our results. Section VI concludes.

## II. Background

There is some existing evidence for Markov-switching nonlinear dynamics in U.S. real GDP growth. Specifically, Morley and Piger (2012) test for nonlinearity using the procedure developed in Carrasco, Hu, and Ploberger (2014) and find support for the Markov-switching model in Kim et al. (2005) that captures U-shaped recessions, but not for the model in Hamilton (1989) that captures L-shaped recessions. However, the tests are applied using data over the sample period of 1947–2006 and so do not include the Great Recession. More recently, Morley and Panovska (2019) conduct tests for nonlinearity using data for a number of countries and find similar results to Morley and Piger (2012) of greater support for a Markov-switching model with U-shaped recessions than L-shaped recessions. For the U.S. data, they find support for nonlinearity when allowing for an estimated slowdown in trend growth in 2000Q2 based on Bai and Perron's (1998, 2003) testing procedures.[2] While it is not straightforward to apply the Carrasco et al. (2014) testing procedure to our model, we note that Eo and Kim (2016) are able to reject simpler models with only one type of recession in favor of more heterogeneity in business cycle regimes using Bayesian methods, thus providing a strong motivation for allowing different types of recessions. Furthermore, we are able to show that the estimated nonlinear dynamics capturing recessions for our model hold up well with more years of data, including enough observations after the end of the Great Recession to discriminate among competing hypotheses about its long-run consequences.

Before presenting the details of our new Markov-switching model, we follow Morley and Panovska (2019) by first considering possible structural breaks in trend growth. We do so by applying Qu and Perron's (2007) testing procedures for multiple structural breaks in mean or variance of quarterly U.S. real GDP growth for the sample period of 1947Q2 to 2018Q4 with 10% trimming at the beginning and the end of the sample and between breakdates.[3] Based on a likelihood ratio test, we find evidence of two breaks, which are estimated to have occurred in 1984Q2 and 2006Q1, respectively, as reported in table 1. These breakdates align with the timing of the Great Moderation widely reported in the literature (Kim & Nelson, 1999b; McConnell & Perez-Quiros, 2000) and the breakdate for the slowdown in trend growth that was also found in Kamber et al. (2018).[4] The structural breaks are significant at the 5% level, and there is no support for an additional break even at a 10% level. Related to

[2]Given the sample period of 1947–2016 and a minimum-length trimming restriction for subsamples of 15% of the total sample when testing for structural breaks, we note that Morley and Panovska (2019) did not consider whether the estimated breakdate corresponds to the Great Recession, while we are able to do so given the availability of a few extra years of data and our different choice of 10% trimming. Also, Bai and Perron's (1998, 2003) procedures allow for a break only in mean, but not variance, unlike the Qu and Perron (2007) procedures considered in our analysis. Note that reporting the breakdate in Morley and Panovska (2019) as 2000Q2 corresponds to the convention of a breakdate being the last period of the previous structural regime.

[3]The raw data for seasonally adjusted quarterly U.S. real GDP and all other series considered in this paper were obtained from the St. Louis Fed database (FRED), and quarterly growth rates were calculated as 100 times the first differences of the natural logarithms of the levels data.

[4]A break in 2006Q1 was also found in Luo and Startz (2014) for Bayesian estimation of an unobserved components model of U.S. real GDP. However, Kim and Chon (2020) show that the results for such a model are more favorable for gradual structural change when the posterior sampler for Bayesian estimation correctly takes correlation between movements in trend and cycle into account.

TABLE 1.—STRUCTURAL BREAKS IN OUTPUT GROWTH

(a) Sequential Break Tests

| # of Breaks | Test Statistic | 5% Critical Value | Estimated Breakdate(s) |
|---|---|---|---|
| 1 | 72.87 | 12.80 | 1984Q2 |
| 2 | 18.76 | 13.96 | 1984Q2, 2006Q1 |
| 3 | 8.77 | 14.84 | 1984Q2, 2000Q2, 2009Q2 |

(b) Mean and Standard Deviation given Two Breaks in 1984Q2 and 2006Q1

| Subsample | Mean | Std. Dev. | 95% Confidence Set for Breakdate |
|---|---|---|---|
| 1 | 0.89 | 1.16 | |
| 2 | 0.80 | 0.49 | [1982Q1, 1987Q1] |
| 3 | 0.41 | 0.59 | [1991Q3, 2011Q3] |

Five percent critical values are from Qu and Perron (2007). Ninety-five percent confidence sets are based on the inverted likelihood ratio approach in Eo and Morley (2015).

the Great Moderation and our Markov-switching model, we note that a larger variance for output growth before 1984Q2 could potentially be related to a more frequent realization of recessions before the mid-1980s. In particular, the postwar U.S. economy experienced eight recessions between 1947 and 1984 (37 years), but only three recessions between 1985 and 2018 (34 years). Thus, we will be able to use our Markov-switching model to check whether this estimated structural break is due to the less frequent realization of recessions since 1984 or a reduction in residual volatility.

Table 1 also reports estimates for the mean and standard deviation of output growth based on the estimated breakdates, along with the confidence sets for the breakdates. The confidence set for the first breakdate covers a reasonably short interval of 1982Q1 to 1987Q1, while the confidence set for the second breakdate is wider and ranges from 1991Q3 to 2011Q3, where the latter date represents the last possible breakdate given 10% trimming. The estimated breakdate of 2006Q1 is consistent with the date for the growth slowdown in Fernald et al. (2017), and they argue that it reflects slow growth of total factor productivity and a decline in labor force participation that are unrelated to the financial crisis and the Great Recession.

For the first estimated break in 1984Q2, a likelihood ratio test of no change in mean suggests that the break corresponded to a change in variance only, with the sample standard deviation of output growth dropping by more than 50%. The average growth rates before and after the first estimated breakdate of 1984Q2 are very close to each other at 0.89 and 0.80, respectively, in contrast to the average growth rate of 0.41 after the second breakdate of 2006Q1. We note that the decline in average growth since 2006Q1 could be related to the realization of a particularly severe recession between 2007 and 2009. Thus, we will also use our Markov-switching model to check whether this estimated structural break is due to the Great Recession or a more sustained decline in trend growth. We will also determine whether explicitly accounting for nonlinear dynamics affects the precision of inferences about the timing of structural breaks.

## III. Model

We develop a new univariate Markov-switching model of real GDP growth that accommodates two different types of recessions. In particular, the model builds on the Markov-switching models in Hamilton (1989) and Kim et al. (2005), which assume all recessions have the same dynamics by allowing for two distinct contractionary regimes: (a) an L-shaped regime with permanent effects on the level of output, as in Hamilton (1989), and (b) a U-shaped regime with temporary effects, corresponding to a restricted version of the model in Kim et al. (2005) that is related to Kim and Nelson (1999a). The idea of allowing for different contractionary regimes is strongly motivated by Eo and Kim (2016), who find a Markov-switching model with time-varying, regime-dependent mean growth rates that depend on each other across booms and recessions fits the U.S. data better than the simpler Markov-switching models in Hamilton (1989) and Kim et al. (2005).

Extending the specification in Kim et al. (2005), we assume that output growth, $\Delta y_t$, has the following time-varying mean based on three regimes.

$$\Delta y_t = \mu_0 + \mu_1 \times \mathbf{1}(S_t = 1) + \mu_2 \times \mathbf{1}(S_t = 2) + \lambda_2 \times \sum_{k=1}^{m} \mathbf{1}(S_{t-k} = 2) + e_t, \quad (1)$$

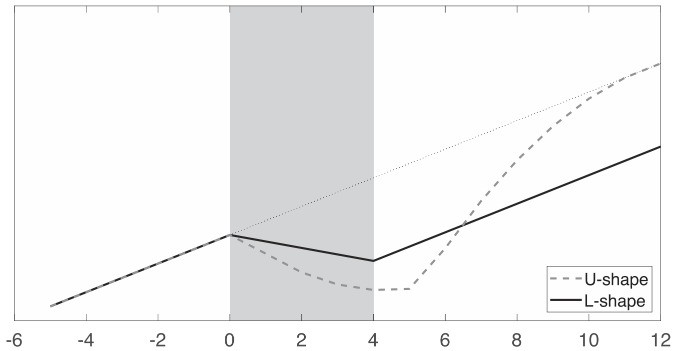
where $\mathbf{1}(\cdot)$ is an indicator function, $S_t$ is a latent Markov-switching state variable that takes on discrete values such that $S_t = 0$ for the expansionary regime, $S_t = 1$ for the L-shaped contractionary regime, and $S_t = 2$ for the U-shaped contractionary regime according to transition probabilities $Pr[S_t = j|S_{t-1} = i] = p_{ij}$ for $i, j = 0, 1, 2$, and $e_t \sim N(0, \sigma^2)$. For simplicity and following the empirical results in Hamilton (1989), Kim et al. (2005), Morley and Piger (2012), and Eo and Kim (2016), we abstract from autoregressive dynamics in linear shocks by assuming that $e_t$ is serially uncorrelated.[5]

To identify the contractionary regimes as corresponding to two different types of recessions, we assume that the economy does not switch between contractionary regimes without going through an expansionary regime first. This sequencing of regimes is imposed using the following restrictions on the regime transition probabilities: $p_{12} = 0$ and $p_{21} = 0$. Meanwhile, the $\lambda_2$ parameter is the key distinctive feature of the U-shaped contractionary regime in equation (1) because it allows for a bounceback effect that generates an asymmetric output gap, as in Morley and Piger (2012) and Morley and Panovska (2019).[6] To clearly identify this regime as distinct from the L-shaped regime that only has permanent effects on

[5]Specifically, these earlier papers find that linear autoregressive dynamics in the residual are not particularly important, once allowing for a Markov-switching mean. However, it is important to note that the statistical evidence for Markov-switching nonlinearity discussed in section II allows for AR(2) dynamics in output growth under the null of linearity.

[6]Possible sources of an asymmetric output gap are capacity constraints, monopoly power, asymmetric wage and price adjustments, collateral

FIGURE 1.—ILLUSTRATION OF DIFFERENT TYPES OF RECESSIONS




The shaded area denotes the contractionary regime.

the level of output by construction, we impose the restriction $\mu_2 + m \times \lambda_2 = 0$.[7] This restriction implies that following the realization of $S_t = 2$, the bounceback effect $m \times \lambda_2$ exactly cancels out the contractionary effect from $\mu_2$ such that the U-shaped regime has only temporary effects on the level of output, as in the Markov-switching model in Kim and Nelson (1999a) but distinct from the model in Kim et al. (2005), which does not impose this restriction.[8]

Figure 1 illustrates how the two contractionary regimes create different types of recessions in terms of their long-run effects on the level of output. In particular, we plot the path of output implied by the model in equation (1) before, during, and after the occurrence of a contractionary regime. Motivated by the estimates for our benchmark model presented in the next section, we set the length of the postrecession bounceback effect to $m = 5$ quarters and the model parameters to be $\mu_0 = 0.91$ for the expansionary regime, $\mu_1 = -1.32$ for the L-shaped regime, and $\mu_2 = -2.10$ for the U-shaped regime (thus implying $\lambda_2 = 0.42$). For clarity in seeing the relative impact of the two different regimes, we abstract from the linear $e_t$ shocks when calculating the path of output. We assume that the economy enters a contractionary regime at time $t = 0$ that lasts for four quarters for the L-shaped regime and five quarters for the U-shaped regime. The longer duration for the U-shaped regime is motivated by a higher continuation probability in the next section.

constraints, aggregation of microeconomic shocks, and underlying asymmetric shocks. See Friedman (1964, 1993); DeLong and Summers (1988); Auroba, Bocola, and Schorfheide (2013); Guerrieri and Iacoviello (2016); Baqaee and Farhi (2019); and Dupraz, Nakamura, and Steinsson (2019), among many others, for more information on these theories of business cycle asymmetry. Also see Morley (2009, 2019) for surveys of the broader literature on business cycle asymmetry.

[7]Typically with Markov-switching models, it is necessary to place labeling restrictions such as $\mu_1 < 0$ and $\mu_2 < 0$ to identify the model. However, because there is no bounceback effect when $S_t \neq 2$, the U-shaped contractionary regime turns out to be uniquely identified given only the restriction on $\lambda_2$ and the restrictions on the transition probabilities. Thus, we place no restrictions on the other parameters in equation (1).

[8]In addition to our consideration of a latent Markov-switching state variable instead of predetermined NBER dates, this restriction on the bounceback effect is another key distinction from Huang et al. (2016), who allow for possible permanent effects with their U-shaped regime, as in Kim et al. (2005), in addition to assuming permanent effects with their L-shaped regime.

However, because the bounceback effect takes hold as the U-shaped regime persists and flattens out the path of output, there is only an outright recession in the level of output for four quarters in both cases. After the flat path for output for the one additional quarter of the U-shaped regime, the economy grows quickly and eventually recovers to its prerecession path. In this sense, the recession has no permanent effect on the level of output, and its path traces out what looks like a tilted and elongated U. By contrast, for the L-shaped regime, the absence of a bounceback effect means that the economy contracts sharply in the recession and never recovers to its prerecession path, growing only at the usual expansionary rate when the recession is over. Thus, this recession has a permanent effect on the level of output, and its path traces out what looks like a tilted L.

## IV. Benchmark Results

Estimation is conducted via maximum likelihood, where the conditional likelihood function given the length of the postrecession bounceback effect $m$ is evaluated based on the filter presented in Hamilton (1989) keeping track of $3^{m+1}$ states in each period. The estimate of the discrete-value parameter $m$ is also chosen to maximize the likelihood by considering the profile likelihood for $m$ across a set of different possible values (capped, for computational feasibility, at $m = 7$, corresponding to 6,561 possible states to keep track of in estimation). Because the estimates of the other parameters are calculated using the conditional likelihood function, reported standard errors based on numerical second derivatives do not reflect sampling uncertainty about $\hat{m}$.

To incorporate the possible structural breaks found in section II into the benchmark version of our model, we modify the basic model in equation (1) as follows:

$$\Delta y_t = \mu_0 + \delta \times \mathbf{1}(t > \tau) + \mu_1 \times \mathbf{1}(S_t = 1) + \mu_2 \times \mathbf{1}(S_t = 2) + \lambda_2 \times \sum_{k=1}^{m} \mathbf{1}(S_{t-k} = 2) + e_t, \quad (2)$$

where $e_t \sim N(0, \sigma_t^2)$, with $\sigma_t^2 = \sigma_{\upsilon 0}^2 \times \mathbf{1}(t \leq \tau_\upsilon) + \sigma_{\upsilon 1}^2 \times \mathbf{1}(t > \tau_\upsilon)$. Based on the findings in section II, we assume the breakdates $\tau = 2006Q1$ for trend growth and $\tau_\upsilon = 1984Q2$ for residual volatility in our benchmark model. If the breaks found in section II actually reflected the severe recession in 2007–2009 and the less frequent realization of recessions in the second half of the sample period, then the estimate for $\delta$ should be small and $\sigma_{\upsilon 1}^2$ should be similar to $\sigma_{\upsilon 0}^2$. However, incorporating these structural breaks allows for a permanent trend growth slowdown and a reduction in the volatility of linear shocks in the second half of the sample period if these phenomena remain relevant even when accounting for Markov-switching dynamics.

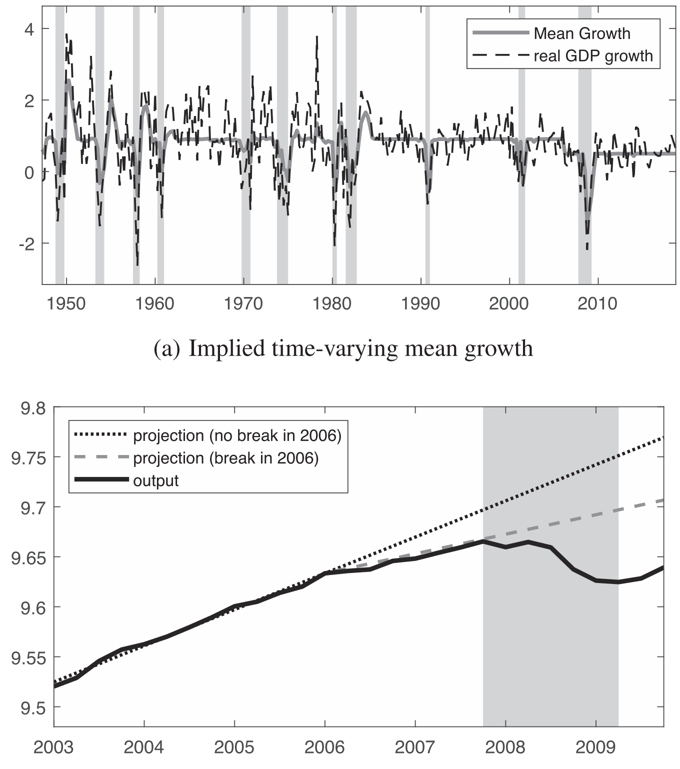
Table 2 reports maximum likelihood estimates for the benchmark model. The implied growth rates are $\hat{\mu}_0 + \hat{\mu}_1 < 0$

Table 2.—Parameter Estimates for the Benchmark Model

| Parameter | Estimate | Standard Error |
|---|---|---|
| $p_{01}$ | 0.03 | 0.01 |
| $p_{02}$ | 0.02 | 0.01 |
| $p_{11}$ | 0.66 | 0.17 |
| $p_{22}$ | 0.73 | 0.13 |
| $\mu_0$ | 0.91 | 0.05 |
| $\mu_1$ | −1.32 | 0.27 |
| $\mu_2$ | −2.10 | 0.29 |
| $\lambda_2$ | 0.42 | 0.06 |
| $\delta$ | −0.41 | 0.08 |
| $\sigma_{v0}$ | 0.90 | 0.07 |
| $\sigma_{v1}$ | 0.42 | 0.03 |
| $m$ | 5 | |
| log-lik | −317.35 | |

The benchmark model is given by equation (2) with $\tau = 2006Q1$, and $\tau_v = 1984Q2$. Estimates are reported for both $\mu_2$ and $\lambda_2$ even though they are jointly estimated using the restriction $\mu_2 + m \times \lambda_2 = 0$.

Figure 2.—Mean Growth and Projected Output




(a) Implied time-varying mean growth

(b) Projected log output around the Great Recession

The shaded areas denote NBER recession dates.

for the L-shaped regime and $\hat{\mu}_0 + \hat{\mu}_2 < 0$ for the U-shaped regime, indicating that both regimes are indeed contractionary, even though this was not imposed in estimation. The estimated transition probabilities suggest that expansions are much more persistent than either type of recession, like the NBER reference cycle. In particular, the implied continuation probability of the expansionary regime $1 - \hat{p}_{01} - \hat{p}_{02}$ is 0.96, with an expected duration of 23 quarters, while the expected duration is three quarters for the L-shaped regime and four quarters for the U-shaped regime. Residual volatility is estimated to have dropped by more than half in 1984Q2, suggesting the Great Moderation was not simply due to less frequent realization of recessions. Meanwhile, the estimated reduction in trend growth in 2006Q1 of −0.41 is very close to the reduction of −0.39 found with the Qu and Perron (2007) analysis in section II, suggesting that lower average growth since 2006 was also not simply due to the realization of a severe recession. The estimated length of the postrecession bounceback effect is five quarters, although we note that other parameter estimates are very similar for $m = 6$, which was the length considered in Kim et al. (2005).[9]

Figure 2 plots the implied time-varying mean from the benchmark model using the filtered estimate $E[\mu_t|\Omega_t]$, where $\mu_t \equiv \Delta y_t - e_t$ and $\Omega_t \equiv (\Delta y_1, \Delta y_2, \ldots, \Delta y_t)$. Closely tracking realized real GDP growth and reflecting $\hat{\delta} = -0.41$, the time-varying mean declines abruptly after 2006Q1, with this slowdown in trend growth explaining the weak recovery of the U.S. economy following the Great Recession.[10] It is also clear that not accounting for a break in trend growth in 2006Q1 would have resulted in persistently downward-biased forecast errors even after the Great Recession.

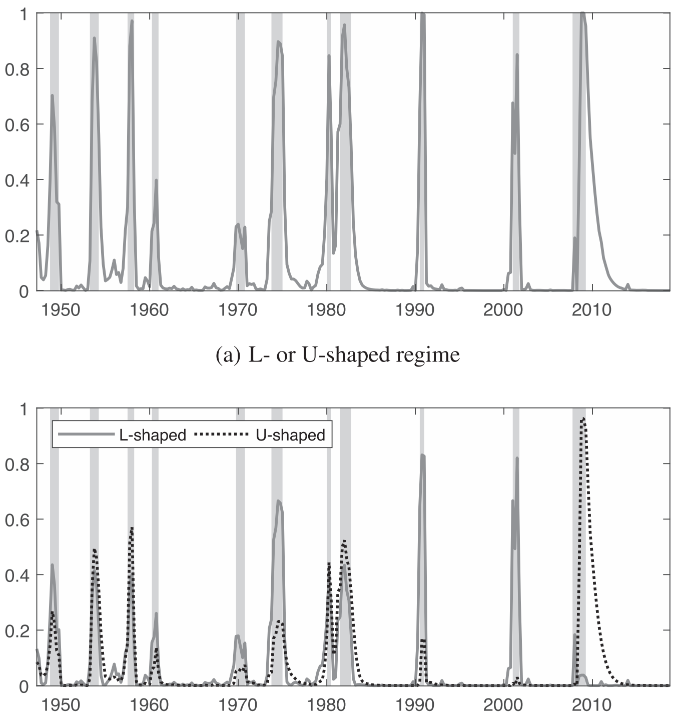
To help illustrate the magnitude of the trend break in 2006Q1, figure 2 also plots projections from $t = 2006Q1$ for future log output $E[y_{t+h}|\Omega_t]$, $h > 0$, both accounting for and not accounting for the structural break. The dotted line shows the projection of log output without accounting for the structural break, which diverges markedly from realized log output (solid line) even before the Great Recession. The dashed line shows the projection accounting for the structural break and clearly supports the idea that the decline in trend growth began in 2006 prior to the onset of the Great Recession. Notably, given the natural log scale, the difference by the end of the Great Recession corresponds to more than 5% of the level of real GDP in 2006Q1.

Figure 3 reports the smoothed probabilities of being in a contractionary regime at time $t$. The top panel plots the probability of being in one or the other regime, calculated from the sum of the probabilities of being in the L-shaped regime and the U-shaped regime using $Pr[t = \text{contraction}|\Omega_T] \equiv Pr[S_t = 1|\Omega_T] + Pr[S_t = 2|\Omega_T]$. This probability closely matches the timing of NBER recessions. In particular, for nine of the eleven NBER recessions in the sample, the smoothed probability is well above 50% for most of a given recession. The bottom panel of figure 3 plots the underlying smoothed probabilities of the L-shaped and U-shaped regimes separately. Considering their relative contribution to the overall

[9]For comparison, the log-likelihood values for $m = 4, 6, 7$ are −318.59, −317.65, and −318.68, respectively.

[10]The top panel of figure 2 looks similar to the estimated time-varying mean in Eo and Kim (2016) for a Markov-switching model with time-varying, regime-dependent mean growth rates that depend on each other across booms and recessions and also allowing for possible structural change in trend growth. Our relatively simple model captures differences in mean growth for each recession and expansion based on whether the contractionary regime is L or U shaped, with mean growth in a recession related to mean growth in the subsequent expansion.

FIGURE 3.—PROBABILITIES OF CONTRACTIONARY REGIMES

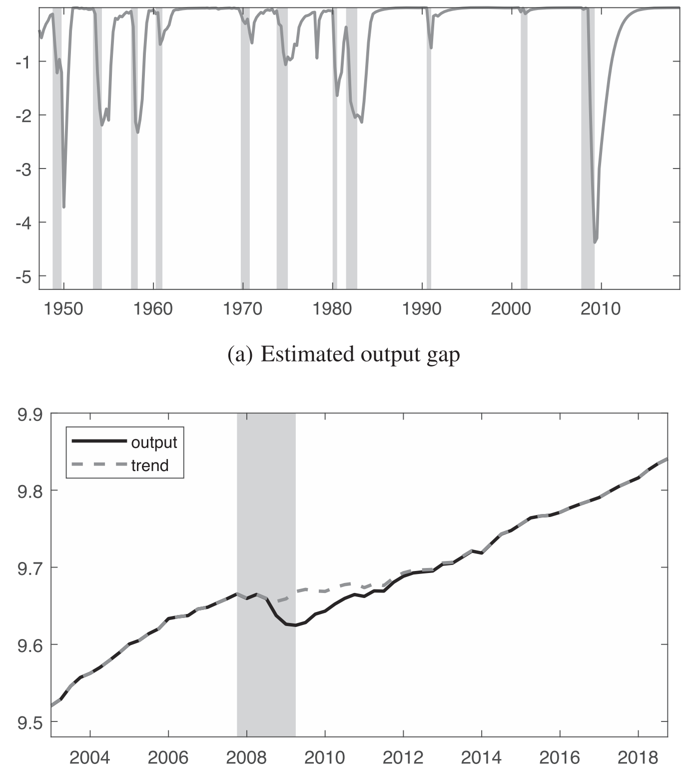



(a) L- or U-shaped regime

(b) L- and U-shaped regimes

The shaded areas denote NBER recession dates.

FIGURE 4.—OUTPUT GAP AND TREND




(a) Estimated output gap

(b) Estimated trend path around the Great Recession

The shaded areas denote NBER recession dates.

probability of a contractionary regime, these probabilities suggest that the 1973–1975, 1990–1991, and 2001 recessions in particular can be classified as L shaped, while the 2007–2009 recession can be largely classified as U shaped, with only the small probability of an L-shaped regime at the beginning of the recession implying any level hysteresis effects. The less definitive classification of the other recessions suggests they may exhibit more of a partial recovery, as found for the estimated bounceback model in Kim et al. (2005).

It might seem surprising that the Great Recession is classified as being U shaped given the conventional view that recessions associated with financial crises have large permanent effects on the level of economic activity.[11] Also, mean growth in figure 2 does not display the same surge after the Great Recession as occurred following other recessions with a sizable probability of being U shaped. The econometric explanation for this finding is that the probability corresponding to a U-shaped regime in figure 3 remains elevated for a substantial period of time after the trough date established by the NBER for the Great Recession.[12] This could be related to a prolonged weak labor market ("jobless recovery") following the recession. Also, the zero-lower-bound on interest rates restricted the ability of monetary policy to help stimulate a strong recovery immediately after the recession. Thus, the relatively restrained mean growth following the Great Recession could be related to a large, persistent negative output gap that dissipates very slowly.

To estimate the output gap implied by our model, we adopt the regime-dependent steady-state (RDSS) generalization of the Beveridge and Nelson (1981) decomposition for Markov-switching processes developed in Morley and Piger (2008). This approach involves constructing long-horizon forecasts conditional on sequences of regimes and then marginalizing over the distribution of the unknown regimes. Unlike the traditional Beveridge-Nelson decomposition, there is no implicit assumption that the cycle is unconditionally mean zero, and we choose the expansionary regime as having a mean-zero transitory component.[13]

Figure 4 plots the estimated output gap from the RDSS decomposition implied by the benchmark model. The large, negative movements in the output gap closely match up with some of the NBER-dated recessions. However, because an L-shaped contractionary regime is assumed to only affect the trend, the large, negative movements in the output gap

[11] See, for example, Cerra and Saxena (2008), Reinhart and Rogoff (2009), and Jordà, Schularick, and Taylor (2017).

[12] As illustrated in figure 1, a U-shaped regime can imply flat growth after the end of a recession if the regime persists long enough before the eventual recovery to the prerecession path.

[13] See Morley and Piger (2008) for a full discussion of this choice and Morley and Piger (2012) for a justification of choosing the expansionary regime as having a mean-zero transitory component.

correspond primarily to the recessions with a high probability of being U shaped. In terms of the Great Recession, the negative output gap opens up later than the NBER peak date of 2007Q4, corresponding to when the probability of U-shaped regime spikes up in figures 3. As the bottom panel of figure 2 makes clear, the reason for this different timing is that the level of real GDP does not decline sharply until the second half of 2008, although real GDP did not grow at its usual expansionary rate in the first half of 2008, even accounting for the structural break in trend growth. This delayed timing of the severe contraction for the Great Recession is distinct from the behavior of real GDP in previous recessions and could perhaps reflect a misattribution by the NBER of a particularly lackluster manifestation of weak trend growth during the first half of 2008 as being part of the recession phase.[14] Meanwhile, the output gap remains persistently negative long after the NBER trough date, corresponding to only a very slow recovery in the level of output.

Figure 4 also plots log output and the estimated trend path from the RDSS decomposition around the Great Recession. The magnitude and persistence of the output gap following the recession are clear from this plot. In particular, the implied negative output gap is not estimated to fully close until around 2012. Because the closure of the output gap is so slow, there is no apparent surge in output growth following the recession in the top panel of figure 2. However, it is important to note that this estimated dynamic of a persistent negative output gap is clearly distinctly identified from an L-shaped recession that only alters the level of trend output. If we consider a modification of our model to impose that the Great Recession was L shaped and not U shaped, such as was found in Huang et al. (2016) using NBER dates for the regimes, the fit of the model noticeably deteriorates, with the log likelihood dropping to −319.61 from −317.35 for our benchmark model.[15] The deterioration of fit appears to be due to a failure to capture the rounded U shape of the recession as it approaches its trough and an eventual gradual recovery of output to a trend path that are both evident in the bottom panel of figure 4.

## V. Robustness

In this section, we consider some alternatives to our benchmark model in order to investigate the robustness and interpretation of our results. First, we estimate two models that allow us to consider whether there are really different types of recessions in terms of their permanent effects on the level of output. Second, we estimate a model using output per capita and examine the possible role of demographic factors in driving our results. Third, we directly estimate breakdates for the structural breaks in trend growth and residual volatility as additional parameters in the model rather than assuming the estimated breakdates from section II. Fourth, we check whether our inferences about the Great Recession are robust to alternative assumptions about the nature of structural change in trend growth and the length of the postrecession bounceback effect.

### A. *Are There Really Different Types of Recessions?*

To consider whether there are actually different types of recessions, we estimate two alternative models. The first model is more general than our benchmark specification in that it allows for a possible bounceback effect in the first contractionary regime in addition to the assumed full recovery in the second contractionary regime,

$$\begin{aligned}\Delta y_t = {} & \mu_0 + \delta \times \mathbf{1}(t > \tau) \\ & + \mu_1 \times \mathbf{1}(S_t = 1) + \lambda_1 \times \sum_{k=1}^{m} \mathbf{1}(S_{t-k} = 1) \\ & + \mu_2 \times \mathbf{1}(S_t = 2) + \lambda_2 \times \sum_{k=1}^{m} \mathbf{1}(S_{t-k} = 2) + e_t, \quad (3)\end{aligned}$$

where the possibility that $\lambda_1 \neq 0$ makes the model more general than equation (2). Unlike $\lambda_2$, which is constrained such that $\mu_2 + m \times \lambda_2 = 0$, we leave $\lambda_1$ unrestricted in estimation. Thus, the general model nests our benchmark model if $\hat{\lambda}_1 = 0$. In principle, it also nests the possibility that there are only U-shaped recessions with full recoveries if $\hat{\mu}_1 = \hat{\mu}_2$ and $\hat{\lambda}_1 = \hat{\lambda}_2$, although the regime transition probabilities would not be identified in such a case. The second model is a restricted version of the general model in equation (3) with only one contractionary regime and corresponds to the original bounceback model in Kim et al. (2005):

$$\begin{aligned}\Delta y_t = {} & \mu_0 + \delta \times \mathbf{1}(t > \tau) + \mu_1 \times \mathbf{1}(S_t = 1) \\ & + \lambda_1 \times \sum_{k=1}^{m} \mathbf{1}(S_{t-k} = 1) + e_t, \quad (4)\end{aligned}$$

where, again, we leave $\lambda_1$ unrestricted in estimation and only need to estimate regime transition parameters $p_{01}$ and $p_{11}$. This restricted model nests the possibility that there are only L-shaped recessions if $\hat{\lambda}_1 = 0$. For both alternative models, $e_t \sim N(0, \sigma_t^2)$ is specified as in equation (2) to allow for a structural break in residual volatility. The breakdates are also the same as in the benchmark model: $\tau$= 2006Q1 and $\tau_v$ = 1984Q2. For direct comparability to our benchmark model, we set $m = 5$ rather than estimate it.

Table 3 reports maximum likelihood estimates for the two alternative models in equations (3) and (4). For the general model, the estimate for the additional parameter is $\hat{\lambda}_1 < 0$, implying prolonged slow growth following an L-shaped

[14]Instead, the weak growth may be related to a typical end-of-expansion overhiring phenomenon (Gordon, 2003) that lowered productivity prior to the sharp contraction in the second half of 2008.

[15]To estimate a restricted model that imposes the Great Recession is an L-shaped regime, we set the parameters for the expansionary and U-shaped regimes to temporarily take on implausible values for the duration of the NBER dates corresponding to the Great Recession.

TABLE 3.—PARAMETER ESTIMATES FOR ALTERNATIVE MODELS AND FOR OUTPUT GROWTH PER CAPITA

| | General Model | | Restricted Model | | Per Capita | |
|---|---|---|---|---|---|---|
| Parameter | Estimate | S.E. | Estimate | S.E. | Estimate | S.E. |
| $p_{01}$ | 0.03 | 0.02 | 0.05 | 0.02 | 0.03 | 0.01 |
| $p_{02}$ | 0.02 | 0.01 | | | 0.02 | 0.01 |
| $p_{11}$ | 0.68 | 0.14 | 0.80 | 0.07 | 0.75 | 0.09 |
| $p_{22}$ | 0.73 | 0.13 | | | 0.67 | 0.13 |
| $\mu_0$ | 0.94 | 0.05 | 0.94 | 0.05 | 0.64 | 0.04 |
| $\mu_1$ | −1.02 | 0.22 | −1.24 | 0.15 | −1.29 | 0.13 |
| $\mu_2$ | −2.08 | 0.28 | | | −2.09 | 0.22 |
| $\lambda_1$ | −0.10 | 0.05 | 0.12 | 0.04 | | |
| $\lambda_2$ | 0.42 | 0.06 | | | 0.42 | 0.04 |
| $\delta$ | −0.39 | 0.08 | −0.48 | 0.08 | −0.40 | 0.07 |
| $\sigma_{v0}$ | 0.90 | 0.07 | 1.00 | 0.07 | 0.87 | 0.06 |
| $\sigma_{v1}$ | 0.41 | 0.03 | 0.43 | 0.03 | 0.40 | 0.03 |
| log-lik | −315.74 | | −323.87 | | −313.69 | |

The general model is given by equation (3), the restricted model is given by equation (4), and the model specification for output growth per capita is the same as the benchmark case in equation (2), with $\tau = 2006Q1$, $\tau_v = 1984Q2$, and $m = 5$ in all three cases. Estimates are reported for both $\mu_2$ and $\lambda_2$ even though they are jointly estimated using the restriction $\mu_2 + m \times \lambda_2 = 0$.

TABLE 4.—TREND GROWTH DECOMPOSITIONS

| | | $\Delta \ln Y_t$ | $\Delta \ln(Y_t/N_t)$ | $\Delta \ln(Y_t/E_t)$ | $\Delta \ln(E_t/N_t)$ | $\Delta \ln N_t$ |
|---|---|---|---|---|---|---|
| Model-implied trend growth | Pre-2006 | 0.91 | 0.64 | 0.45 | 0.13 | |
| | Post-2006 | 0.50 | 0.24 | 0.23 | 0.03 | |
| | Reduction | −0.41 | −0.40 | −0.22 | −0.10 | |
| Average growth | Pre-2006 | 0.86 | 0.52 | 0.47 | 0.05 | 0.34 |
| | Post-2006 | 0.41 | 0.16 | 0.23 | −0.08 | 0.25 |
| | Reduction | −0.45 | −0.36 | −0.24 | −0.12 | −0.09 |

$Y_t$, $N_t$, and $E_t$ denote output, population, and employment, respectively. Model-implied trend growth rates correspond to estimated growth in the expansionary regime in equation (2). Unlike average growth rates, they do not necessarily add up exactly according to accounting relationships due to differences in estimated regimes for the different variables.

recession rather than any bounceback effect. Given an offsetting smaller magnitude for $\hat{\mu}_1$ and other parameters similar to those in table 2, the implied dynamic effects of the two types of recessions are close to those in the benchmark model.[16] Meanwhile, for the restricted model, the estimates imply only a partial recovery for all recessions given that $\hat{\lambda}_1 < -\hat{\mu}_1/5$. The estimated expected partial recovery looks like an averaging of the estimated effects of the two contractionary regimes for the general model. Notably, the fit of the restricted model is considerably worse, although a likelihood ratio test of the two models would not have a standard distribution. Taken together, though, these results, combined with the different smoothed probabilities for the two regimes in the bottom panel of figure 3, support the existence of different types of recessions in the U.S. economy.

### *B. What Role Do Demographic Factors Play in the Growth Slowdown?*

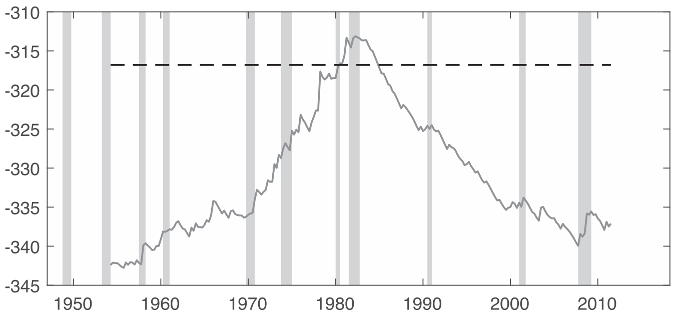
We apply our benchmark model specification with two different types of recessions to output growth per capita instead of overall output growth in order to isolate the effects of population growth on overall trend output growth. Table 3 also reports the estimates for this case. The estimates are strikingly similar to those for output growth presented in table 2. One particularly notable similarity is that the slowdown in trend growth per capita is estimated to be $\hat{\delta} = -0.40$, which is very close to $\hat{\delta} = -0.41$ for the benchmark model. This directly implies that population growth is not responsible for the slowdown in overall trend output growth since 2006 but instead suggests possible roles for productivity and labor force participation (Stock & Watson, 2012; Fernald et al., 2017).

Table 4 reports trend growth decompositions based on basic accounting relationships between the growth rates of output, output per capita, output per employed worker, the employment-population ratio, and population for both before and after a breakdate of 2006Q1.[17] We consider the estimated growth in an expansionary regime implied by our model estimated for the various growth rate series, as well as subsample averages for comparison. Corresponding to the results reported in tables 2 and 3, a lot of the slowdown in overall trend growth can be explained by a reduction in the growth rate of output per capita rather than population growth. Indeed, in terms of estimates for our benchmark model, almost all of the slowdown is accounted for by a reduction in trend growth for output per capita. In terms of the estimates based on subsample averages, most of the slowdown is accounted for in the same way, although we note that the Great Recession has considerable influence on average growth rates since 2006 that is controlled for in our model-based estimates

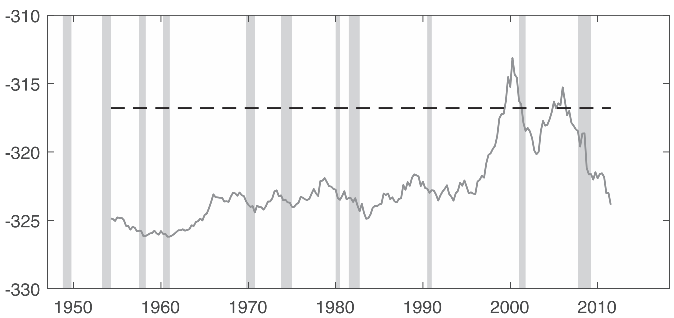
[16]Note that if we constrain $\lambda_1 \geq 0$, the maximum likelihood estimate is exactly $\hat{\lambda}_1 = 0$.

[17]The accounting relationships that inform our trend growth decompositions are $\Delta \ln Y_t \equiv \Delta \ln(Y_t/N_t) + \Delta \ln N_t$, and $\Delta \ln Y_t \equiv \Delta \ln(Y_t/E_t) + \Delta \ln(E_t/N_t) + \Delta \ln N_t$, where $Y_t$, $N_t$, and $E_t$ denote output, population, and employment, respectively.

Figure 5.—Profile Likelihoods for Breakdates




(a) Residual volatility breakdate $\tau_v$




(b) Trend growth breakdate $\tau$

The solid lines plot log-likelihood values for different possible breakdates, conditioning on $\tau = 2000Q2$ for $\tau_v$ and $\tau_v = 1982Q2$ for $\tau$. The dashed horizontal lines correspond to cutoffs for 95% confidence sets based on inverted likelihood ratio tests for a breakdate from Eo and Morley (2015).

of trend growth. For output per capita growth, more of the slowdown can be explained by a reduction in the growth of output per employed worker than by a reduction in the growth of the employment-population ratio. Thus, these results suggest that productivity played a bigger role than demographic factors in explaining the slowdown in overall trend growth.

### C. *What Does Our Model Imply about Timing of Structural Breaks?*

In section II, we estimated breakdates of 1984Q2 and 2006Q1 for output volatility and trend growth, respectively, using Qu and Perron (2007) testing procedures. Based on this result, we assumed these breakdates as known parameters $\tau_v$ and $\tau$ when estimating the benchmark model in section IV. Here, we examine whether inferences about structural breaks are robust to estimating their timing under the assumption that our Markov-switching model captures the dynamics of output growth.

Figure 5 plots profile likelihoods for the breakdates based on the Markov-switching model in equation (2). In particular, the top panel shows the results for the residual volatility breakdate $\tau_v$, and the bottom panel shows the results for the trend growth breakdate $\tau$.[18] The maximum likelihood estimate for the structural break in residual volatility is 1982Q2, which is close to the breakdate of 1984Q2 assumed in our benchmark model. The log-likelihood value for the volatility breakdate of 1982Q2 is −315.28 compared to the value of −317.35 for the benchmark model with the breakdate in 1984Q2. The difference is less than the cutoff value used for constructing a 95% confidence set for a breakdate in Eo and Morley (2015). Therefore, the confidence set for the volatility breakdate includes the benchmark assumption of 1984Q2 obtained from Qu and Perron (2007) procedures in section II. The maximum likelihood estimate for the structural break in trend growth of 2000Q2 is the same breakdate as found in Morley and Panovska (2019) using Bai and Perron (1998, 2003) testing procedures for a shorter sample period. However, 2006Q1 is a local mode for the profile likelihood and cannot be rejected using the cutoff value for constructing a 95% confidence set for a breakdate in Eo and Morley (2015). Furthermore, the last date in the 95% confidence set is 2006Q2, and we find no support for an additional structural break in trend growth. Thus, compared to the results for the Qu and Perron (2007) procedures, our Markov-switching model sharpens inferences about the timing of a structural break in trend growth and allows us to formally reject that the trend growth slowdown occurred either during or after the Great Recession.

If the structural break in trend growth actually occurred in 2000Q2, as implied by the highest mode in the bottom panel of figure 5, it is even clearer than with an estimate in 2006Q1 that it is unrelated to the Great Recession or the forces that led to the financial crisis. At the same time, it is possible that the spike in the likelihood in 2000Q2 is somehow related to in-sample overfitting of the slow growth right before and during the 2001 recession. Looking back at figure 2, it is possible to see how a trend growth slowdown could capture the weak output growth between 2000 and 2002 without having to capture the 2001 recession as being due to a contractionary regime shift. However, a trend growth slowdown in 2000Q2 would also appear to imply a positive bias in forecast errors for the model in the mid-2000s before the shift back down in mean growth in 2006Q1. Next, we further investigate the possibility of overfitting, as well as robustness of our inferences about the Great Recession to different assumptions about structural change, including that a break in trend growth may have occurred in 2000Q2.

[18]The profile likelihoods are calculated as log-likelihood values for different possible breakdates, conditioning on the maximum likelihood estimate of a trend growth break in 2000Q2 in the case of $\tau_v$ and the maximum likelihood estimate of a residual volatility break in 1982Q2 in the case of $\tau$ and maximizing the other parameters out of the likelihood for each possible breakdate. We condition on the maximum likelihood estimate for the other breakdate for computational simplicity, although we have confirmed these are maximum likelihood estimates by calculating the likelihood for a grid of possible breakdates in residual volatility between 1979Q3 and 1987Q1 and breakdates in trend growth between 1997Q2 and 2012Q2. The profile likelihoods that maximize all other parameters including the other breakdate out of the likelihood (and assuming the conditional maximum likelihood estimate of the other breakdate is always in the included range) are almost identical to the profile likelihoods presented in figure 5 for the ranges covered by the grid.

TABLE 5.—PARAMETER ESTIMATES UNDER DIFFERENT ASSUMPTIONS ABOUT STRUCTURAL CHANGE

| | No Break | | Break in 2000Q2 | | Dynamic Demeaning | |
|---|---|---|---|---|---|---|
| Parameter | Estimate | S.E. | Estimate | S.E. | Estimate | S.E. |
| $p_{01}$ | 0.02 | 0.01 | 0.02 | 0.01 | 0.03 | 0.02 |
| $p_{02}$ | 0.02 | 0.01 | 0.02 | 0.01 | 0.02 | 0.01 |
| $p_{11}$ | 0.65 | 0.21 | 0.67 | 0.15 | 0.74 | 0.10 |
| $p_{22}$ | 0.73 | 0.12 | 0.72 | 0.13 | 0.73 | 0.12 |
| $\mu_0$ | 0.77 | 0.04 | 0.94 | 0.05 | 0.09 | 0.04 |
| $\mu_1$ | −1.48 | 0.29 | −1.55 | 0.21 | −1.19 | 0.15 |
| $\mu_2$ | −2.03 | 0.28 | −2.10 | 0.29 | −2.11 | 0.27 |
| $\lambda_2$ | 0.41 | 0.06 | 0.42 | 0.06 | 0.42 | 0.05 |
| $\delta$ | | | −0.39 | 0.07 | | |
| $\sigma_{\upsilon 0}$ | 0.93 | 0.07 | 0.90 | 0.07 | 0.87 | 0.06 |
| $\sigma_{\upsilon 1}$ | 0.46 | 0.03 | 0.42 | 0.03 | 0.43 | 0.03 |
| log-lik | −329.10 | | −315.05 | | −321.21 | |

The model specification is the same as the benchmark case in equation (2) with $\tau_\upsilon = 1984Q2$ and $m = 5$, but with the following assumptions for trend growth: (a) no break; (b) $\tau = 2000Q2$; and (c) structural change is gradual and can be captured by a backward-looking rolling forty-quarter average growth rate. Estimates are reported for both $\mu_2$ and $\lambda_2$ even though they are jointly estimated using the restriction $\mu_2 + m \times \lambda_2 = 0$.



### D. *How Robust Are Inferences about the Great Recession?*

To the extent that there is uncertainty about the timing of an apparent structural break in trend growth or whether it is even best characterized by a single abrupt break (Stock & Watson, 2012; Eo & Kim, 2016; Antolin-Diaz et al., 2017; Kim & Chon, 2020), it is important to investigate the robustness of our inferences regarding the nature of the Great Recession to different assumptions about structural change. To do so, we consider the following alternative cases for trend growth: no break; a break in 2000Q2; gradual change addressed by dynamically demeaning output growth rate using a backward-looking rolling 40-quarter average growth rate, as in Kamber et al. (2018); and gradual change addressed by using weighted-average inferences based on the relative profile likelihood value over all of the possible breakdates, as discussed in more detail below.

Table 5 reports the parameter estimates for our Markov-switching model under the first three assumptions for trend growth of no break, a break in 2000Q2, and gradual change addressed by dynamic demeaning.[19] Notably, for all three of these alternative assumptions, the parameter estimates related to the effects of recessions are highly robust and similar to the estimates for the benchmark model in table 2. Meanwhile, looking at the log-likelihood values, the fit for dynamic demeaning and especially the no break case is worse than in the benchmark case or when allowing for a break in 2000Q2.[20]

For weighted-average inferences to capture possible gradual change, we calculate probabilistic weights over different possible breakdates in trend growth. In particular, using the profile likelihood value for each breakdate, the probabilistic weight for a breakdate $\tau$ is calculated as

$$\hat{\omega}(\tau) \equiv \frac{f(y|\hat{\theta}_\tau;\tau)}{\sum_{t\in[0.1T,0.9T]} f(y|\hat{\theta}_t;t)}, \tag{5}$$

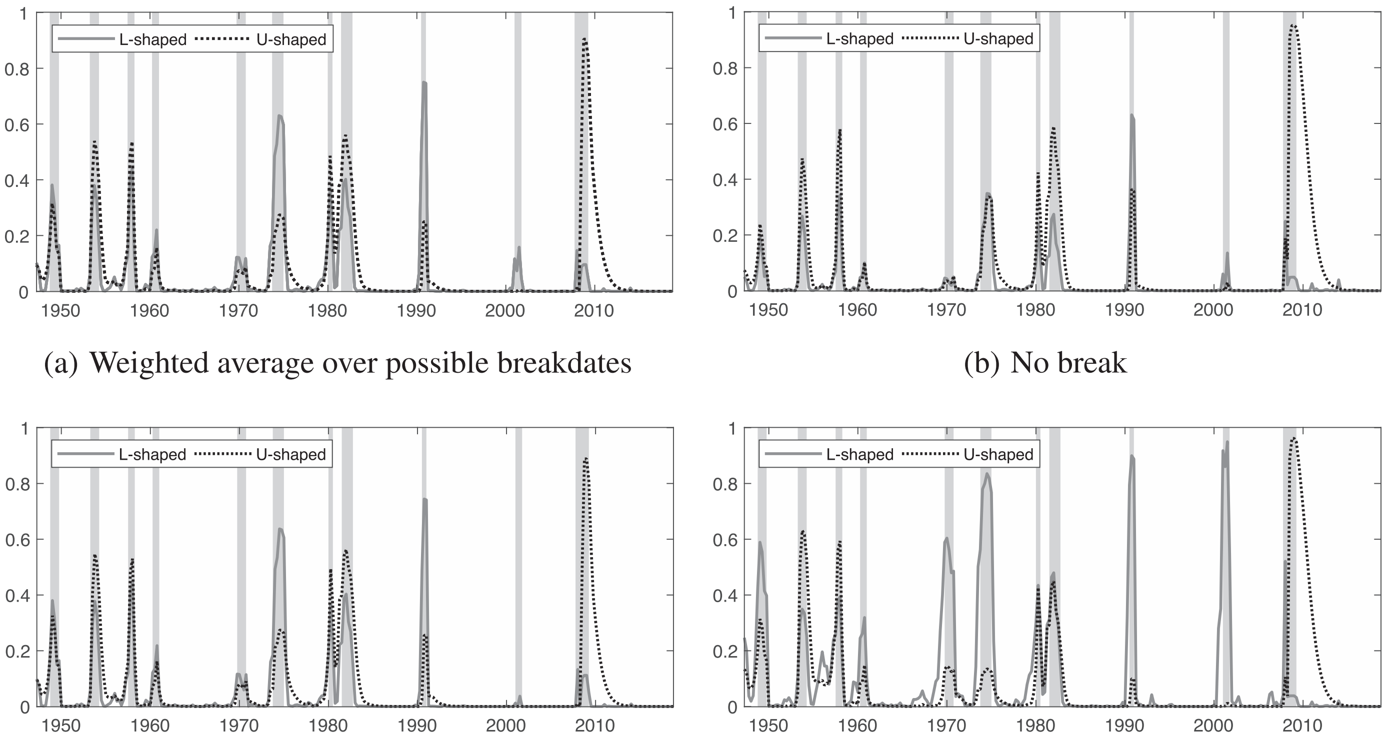
where $f(y|\hat{\theta}_\tau;\tau)$ is the likelihood value for the trend growth breakdate $\tau$ given the model in equation (2) with maximum likelihood estimates $\hat{\theta}_\tau$ for the other parameters conditional on $\tau$, $\tau_\upsilon = 1984Q2$, and $m = 5$. By construction, the sum of the weights over the possible breakdates will equal 1, $\sum_\tau \hat{\omega}(\tau) = 1$. Then, for example, the weighted-average smoothed probability of the regime $j$ at time $t$ given these weights is $\sum_\tau \hat{\omega}(\tau) \times Pr[S_t = j|\Omega_T, \tau]$, where $Pr[S_t = j|\Omega_T, \tau]$ is the smoothed probability of the regime $j$ at time $t$ given the breakdate of $\tau$. Weighted-average inferences inherently lose precision compared to knowing the exact breakdate, but they are potentially robust to multiple breaks in trend growth.

Figure 6 plots smoothed probabilities of the two contractionary regimes for the weighted-average approach, as well as for the different assumptions about structural change reported in table 5. The classification of certain recessions differs across the various cases and sometimes in comparison to the benchmark results in figure 3. For example, it is clear that considering the trend growth break in 2000Q2 means that the 2001 recession would no longer be classified as a contractionary regime, supporting the idea that this timing for the structural break is overfitting the temporary effects of

[19]Following Kamber et al. (2018), dynamic demeaning involves calculating deviations from a slowly moving, time-varying unconditional mean as follows: $\Delta\tilde{y}_t \equiv \Delta y_t - \frac{1}{40}\sum_{j=0}^{39}\Delta y_{t-j}$. We then estimate our Markov-switching model in equation (2) using the dynamically demeaned data $\Delta\tilde{y}_t$ and setting $\delta = 0$, with the residual volatility breakdate still fixed at $\tau_\upsilon =$ 1984Q2 and $m = 5$ for direct comparison to the benchmark case.

[20]Another way to look at model fit is to consider whether the filtered estimates of the residuals display serial correlation. Interestingly, over the subsample from 1984Q3 to 2018Q4, we find that the benchmark model with a break in 2006Q1 has the smallest Ljung-Box Q-statistics of 0.00 (1 lag) and 2.80 (4 lags). The model with a break in 2000Q2 has Q-statistics of 0.09 (1 lag) and 4.16 (4 lags), with the worse fit possibly reflecting a positive bias in forecast errors in the mid-2000s, although we note that consistent with the log likelihood, the model with a break in 2000Q2 has the smallest Q statistics (but very similar to those for the benchmark model) when considering the full sample. The model with dynamic demeaning performs similar to the model with a break in 2000Q2 with Q-statistics of 0.30 (1 lag) and 3.60 (4 lags). Meanwhile, the model with no break has much larger Q-statistics of 2.14 (1 lag) and 12.20 (4 lags), the latter of which is significant at a 5% level. The significant deterioration of fit presumably reflects negative bias in forecast errors since at least 2006Q1 by failing to account for a structural break in trend growth.

FIGURE 6.—PROBABILITIES OF L- AND U-SHAPED REGIMES FOR DIFFERENT CASES




(a) Weighted average over possible breakdates

(b) No break

(c) Break in 2000Q2

(d) Dynamic demeaning

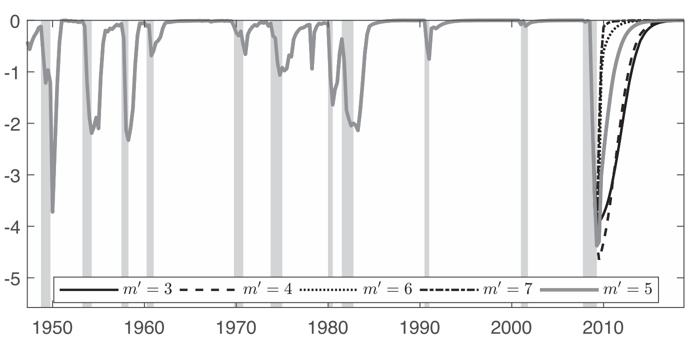
The shaded areas denote NBER recession dates.

the recession on growth rates.[21] However, despite different inferences about some of the recessions, the Great Recession is always classified as being U shaped. Thus, we can be confident that our inferences about the nature of the Great Recession in particular are robust to different assumptions about structural change in trend growth.

As was the case for our benchmark model, the smoothed probabilities in figure 6 directly imply that the Great Recession corresponded to a large, persistent negative output gap. However, the exact persistence of the implied output gap varies considerably across the different alternatives and, in some cases, would suggest that the economy was back at trend even when the unemployment rate remained quite elevated. There is a literature documenting time variation in Okun's law, especially after the Great Recession (Owyang & Sekhposyan, 2012; Grant, 2018). Yet it is important to consider whether the implied persistence of the output gap following the Great Recession is in some way constrained by the structure of our Markov-switching model.

To consider how the structure of our Markov-switching model interacts with inferences about the persistence of the output gap following the Great Recession, we extend the benchmark model in equation (2) to allow for a structural break in the length of the postrecession bounceback effect to

FIGURE 7.—ESTIMATED OUTPUT GAP FOR ALTERNATIVE LENGTHS OF BOUNCEBACK EFFECT FOR THE GREAT RECESSION




The shaded areas denote NBER recession dates.

$m' = 3, 4, 6, 7$ with the Great Recession instead of $m = 5$ for previous recessions. Figure 7 plots the estimated output for different values of $m'$. For $m' = 3, 4$, the estimated output gap is more persistent than in the benchmark case of $m' = m = 5$ and does not close until around 2015. For $m' = 6, 7$, the estimated output gap is less persistent and closes soon after the end of the Great Recession.[22]

[21]The behavior of other variables such as the unemployment rate provides a strong signal that there actually was a recession in 2001.

[22]To understand this counterintuitive result econometrically, note that given a similar estimated negative negative effect of the U-shaped regime $\hat{\mu}_2$ across models with different $m'$, a smaller $m'$ directly implies a larger quarter-by-quarter bounceback effect $\hat{\lambda}'_2$. Because the recovery from the Great Recession was only gradual, this implied strong bounceback is offset

In terms of which $m'$ to choose, we note that the likelihood values for the models with different $m'$ are all very similar to that of the benchmark model, ranging only from $-317.35$ to $-317.34$. However, the relative robustness of the inference about the persistence of the output gap for $m' = 3, 4$ compared to the higher values of $m'$ suggests that the higher values mechanically impose constraints on the estimated persistence of the output gap that the lower values do not.[23] Furthermore, the external consideration of the elevated unemployment rate in the United States above 6% until the middle of 2014 would also seem to support the models with $m' = 3, 4$. The key point, though, is that the inference about the Great Recession being U-shaped is completely robust to different possible values of $m'$.[24]

## VI. Conclusion

We have developed a new Markov-switching model of real GDP growth that accommodates two different types of recessions and allows for structural change in trend growth. Applying our model to U.S. data, we find that, perhaps surprisingly, that the Great Recession was U shaped and did not appear to have any substantial hysteresis effects. Instead, the Great Recession generated a large, persistent negative output gap, with the economy eventually recovering to a lower-growth trend path that, consistent with Fernald et al. (2017), appears to be due to a reduction in productivity growth that began no later than 2006. We highlight that our inferences about the timing of the output growth slowdown are sharpened by our consideration of a time series model that accounts for nonlinear dynamics of recessions. Meanwhile, our inferences about the nature of the Great Recession as generating a persistent negative output gap rather than large hysteresis effects is highly robust to different assumptions regarding the nature of structural change in trend growth.

Our analysis is univariate, and we leave consideration of the implications of our findings for a multivariate setting to future research. However, we note that, similar to the conclusions in Huang and Luo (2018), our estimated output gap can clearly help explain weak inflation in the years immediately after the Great Recession. Our results also suggest that the slow growth of the U.S. economy is likely to persist even when the recession related to the COVID-19 crisis ends and interest rates eventually move back above the zero-lower-bound again. In terms of how the model will classify this latest recession, it is likely to depend on policy responses and require data from the recovery period to discriminate between L- and U-shaped possibilities. Thus, we also leave this to future research.

by the model attributing a high probability that the U-shaped regime persisted well beyond the end of the recession. By contrast, when $m' = 6, 7$, the quarter-by-quarter bounceback effect $\hat{\lambda}'_2$ is smaller and insufficient to offset $\hat{\mu}_2$ in capturing positive but weak growth in real GDP immediately following the recession, but before the full recovery. Thus, in these cases, the model attributes a very low probability that the U-shaped regime persisted beyond the end of the Great Recession.

[23] The likely offsetting benefit of the higher values of $m'$ is that they can capture a smaller quarter-by-quarter bounceback effect.

[24] Because the low probability of an L-shaped regime in the Great Recession could be due to the smaller estimated contractionary effect $\hat{\mu}_1$ compared to $\hat{\mu}_2$ that is evident in figure 1 and table 2, we also considered a model with a structural break in $\mu_1$ to $\mu'_1$ with the Great Recession. The estimated $\hat{\mu}'_1 = -1.92$ does increase the probability that the Great Recession was L shaped, but the probability of U-shaped regime is still higher, with the implied output gap very similar to that for $m' = 7$.

REFERENCES


Antolin-Diaz, Juan, Thomas Drechsel, and Ivan Petrella, "Tracking the Slowdown in Long-Run GDP Growth," this REVIEW 99:2 (2017), 343–356.

Auroba, S. Boragan, Luigi Bocola, and Frank Schorfheide, "Assessing DSGE Model Nonlinearities," *Journal of Economic Dynamics and Control* 83 (2013), 127–142.

Bai, Jushan, and Pierre Perron, "Estimating and Testing Linear Models with Multiple Structural Changes," *Econometrica* 66:1 (1998), 47–78. 10.2307/2998540

——— "Computation and Analysis of Multiple Structural Change Models," *Journal of Applied Econometrics* 18:1 (2003), 1–22. 10.1002/jae.659

Ball, Laurence, "Long-Term Damage from the Great Recession in OECD Countries," *European Journal of Economics and Economic Policies: Intervention* 11:2 (2014), 149–160.

Baqaee, David Rezza, and Emmanuel Farhi, "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem," *Econometrica* 87:4 (2019), 1155–1203. 10.3982/ECTA15202

Beveridge, Stephen, and Charles R. Nelson, "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the 'Business Cycle'," *Journal of Monetary Economics* 7:2 (1981), 151–174. 10.1016/0304-3932(81)90040-4

Blanchard, Olivier, Eugenio Cerutti, and Lawrence Summers, "Inflation and Activity: Two Explorations and Their Monetary Policy Implications," IMF working paper 15/230 (2015).

Blanchard, Olivier J., and Lawrence H. Summers, "Hysteresis and the European Unemployment Problem" (pp. 15–78), in Stanley Fischer, ed., *NBER Macroeconomics Annual*. Cambridge, MA: MIT Press (1986).

Carrasco, Marine, Liang Hu, and Werner Ploberger, "Optimal Test for Markov Switching Parameters," *Econometrica* 82:2 (2014), 765–784. 10.3982/ECTA8609

Cerra, Valerie, and Sweta Chaman Saxena, "Growth Dynamics: The Myth of Economic Recovery," *American Economic Review* 98:1 (2008), 439–457. 10.1257/aer.98.1.439

——— "Booms, Crises, and Recoveries: A New Paradigm of the Business Cycle and Its Policy Implications," IMF working paper 17/250 (2017).

DeLong, J. Bradford, and Lawrence H. Summers, "How Does Macroeconomic Policy Affect Output?" *Brookings Papers on Economic Activity* 2 (1988), 433–480.

Dupraz, Stéphane, Emi Nakamura, and Jón Steinsson, "A Plucking Model of Business Cycles," NBER working paper 26351 (2019).

Eo, Yunjong, and Chang-Jin Kim, "Markov-Switching Models with Evolving Regime-Specific Parameters: Are Postwar Booms or Recessions All Alike?" this REVIEW 98:5 (2016), 940–949. 27986521

Eo, Yunjong, and James Morley, "Likelihood-Ratio-Based Confidence Sets for the Timing of Structural Breaks," *Quantitative Economics* 6:2 (2015), 463–497. 10.3982/QE186

Fernald, John G., Robert E. Hall, James H. Stock, and Mark W. Watson, "The Disappointing Recovery of Output after 2009," *Brookings Papers on Economic Activity* (2017).

Friedman, Milton, "Monetary Studies of the National Bureau" (pp. 7–25), in NBER, ed., *The National Bureau Enters Its 45th Year*, vol. 44. Cambridge, MA: NBER (1964).

——— "The 'Plucking Model' of Business Fluctuations Revisited," *Economic Inquiry* 31 (1993), 171–177. 10.1111/j.1465-7295.1993.tb00874.x

Gordon, Robert J., "Exploding Productivity Growth: Context, Causes, and Implications," *Brookings Papers on Economic Activity* 2003:2 (2003), 207–298. 10.1353/eca.2004.0006

——— "Secular Stagnation: A Supply-Side View," *American Economic Review* 105:5 (2015), 54–59. 10.1257/aer.p20151102
——— *The Rise and Fall of American Growth: The U.S. Standard of Living since the Civil War*. Princeton, NJ: Princeton University Press (2016).
Grant, Angelia L., "The Great Recession and Okun's Law," *Economic Modelling* 69 (2018), 291–300. 10.1016/j.econmod.2017.10.002
Grant, Angelia L., and Joshua C. C. Chan, "A Bayesian Model Comparison for Trend-Cycle Decompositions of Output," *Journal of Money, Credit and Banking* 49:2–3 (2017), 525–552. 10.1111/jmcb.12388
Guerrieri, Luca, and Matteo Iacoviello, "Collateral Constraints and Macroeconomic Asymmetries," *Journal of Monetary Economics* 90 (2016), 28–49.
Guerron-Quintana, Pablo A., and Ryo Jinnai, "Financial Frictions, Trends, and the Great Recession," *Quantitative Economics* 10:2 (2019), 735–773. 10.3982/QE702
Hamilton, James D., "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica* 57:2 (1989), 357–384. 10.2307/1912559
Hansen, Alvin H., "Economic Progress and Declining Population Growth," *American Economic Review* 29:1 (1939), 1–15.
Huang, Yu-Fan, and Sui Luo, "Potential Output and Inflation Dynamics after the Great Recession," *Empirical Economic* 55:2 (2018), 495–517. 10.1007/s00181-017-1293-7
Huang, Yu-Fan, Sui Luo, and Richard Startz, "Are Recoveries All the Same: GDP and TFP?" SSRN working paper 2988263 (2016).
Jordà, Òscar, Moritz Schularick, and Alan M Taylor, "Macrofinancial History and the New Business Cycle Facts," *NBER Macroeconomics Annual* 31:1 (2017), 213–263.
Kamber, Güneş, James Morley, and Benjamin Wong, "Intuitive and Reliable Estimates of the Output Gap from a Beveridge-Nelson Filter," this REVIEW 100:3 (2018), 550–566.
Kim, Chang-Jin, James Morley, and Jeremy Piger, "Nonlinearity and the Permanent Effects of Recessions," *Journal of Applied Econometrics* 20:2 (2005), 291–309. 10.1002/jae.831
Kim, Chang-Jin, and Christian J Murray, "Permanent and Transitory Components of Recessions," *Empirical Economics* 27:2 (2002), 163–183. 10.1007/s001810100097
Kim, Chang-Jin, and Charles R. Nelson, "Friedman's Plucking Model of Business Fluctuations: Tests and Estimates of Permanent and Transitory Components," *Journal of Money, Credit and Banking* 31:3 (1999a), 317–334. 10.2307/2601114
——— "Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-Switching Model of the Business Cycle," this REVIEW 81:4 (1999b), 608–616.
Kim, Chang-Jin, and Jeremy Piger, "Common Stochastic Trends, Common Cycles, and Asymmetry in Economic Fluctuations." *Journal of Monetary Economics*, 49:6 (2002), 1189–1211. 10.1016/S0304-3932(02)00146-0
Kim, Chang-Jin, Jeremy M. Piger, and Richard Startz, "The Dynamic Relationship between Permanent and Transitory Components of US Business Cycles," *Journal of Money, Credit and Bankin* 39:1 (2006), 187–204.
Kim, Jaeho, and Sora Chon, "Why Are Bayesian Trend-Cycle Decompositions of US Real GDP So Different?" *Empirical Economics* 58 (2020), 1339–1354. 10.1007/s00181-018-1554-0
Luo, Sui, and Richard Startz, "Is It One Break or Ongoing Permanent Shocks That Explains US Real GDP?" *Journal of Monetary Economics* 66 (2014), 155–163. 10.1016/j.jmoneco.2014.04.016
McConnell, Margaret M., and Gabriel Perez-Quiros, "Output Fluctuations in the United States: What Has Changed since the Early 1980's?" *American Economic Review* 90:5 (2000), 1464–1476. 10.1257/aer.90.5.1464
Morley, James, "Macroeconomics, Nonlinear Time Series in" (pp. 5325–5348), in R. A. Meyers, ed., *Encyclopedia of Complexity and System Science*. Berlin: Springer (2009).
——— "The Business Cycle: Periodic Pandemic or Rollercoaster Ride?" *International Journal of Economic Policy Studies* 13:2 (2019), 425–431. 10.1007/s42495-019-00023-z
Morley, James C., Charles R. Nelson, and Eric Zivot, "Why Are the Beveridge-Nelson and Unobserved-Components Decompositions of GDP So Different?" this REVIEW 85:2 (2003), 235–243.12943709
Morley, James, and Irina B. Panovska, "Is Business Cycle Asymmetry Intrinsic in Industrialized Economies?" *Macroeconomic Dynamics* 24 (2019), 1–34.
Morley, James, and Jeremy Piger, "Trend/Cycle Decomposition of Regime-Switching Processes," *Journal of Econometrics* 146:2 (2008), 220–226. 10.1016/j.jeconom.2008.08.009
——— "The Asymmetric Business Cycle," this REVIEW 94:1 (2012), 208–221.
Owyang, Michael T., and Tatevik Sekhposyan, "Okun's Law over the Business Cycle: Was the Great Recession All That Different?" *Federal Reserve Bank of St. Louis Review* 94 (September/October 2012), 399–418.
Qu, Zhongjun, and Pierre Perron, "Estimating and Testing Structural Changes in Multivariate Regressions," *Econometrica* 75:2 (2007), 459–502. 10.1111/j.1468-0262.2006.00754.x
Reinhart, Carmen M., and Kenneth S. Rogoff, "The Aftermath of Financial Crises," *American Economic Review* 99:2 (2009), 466–472. 10.1257/aer.99.2.466
Stock, James H., and Mark W. Watson, "Disentangling the Channels of the 2007–2009 Recession," *Brookings Papers on Economic Activity* (Fall 2012).
Summers, Lawrence H., "US Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound," *Business Economics* 49:2 (2014), 65–73. 10.1057/be.2014.13
——— "Demand Side Secular Stagnation," *American Economic Review* 105:5 (2015), 60–65. 10.1257/aer.p20151103