



Check for updates

Citation: Rodríguez-Navarro, A., & Brito, R. (2021). Total number of papers and in a single percentile fully describes research impact—Revisiting concepts and applications. *Quantitative Science Studies*, 2(2), 544–559. https://doi.org/10 .1162/qss_a_00130

DOI: https://doi.org/10.1162/qss_a_00130

Received: 5 November 2020 Accepted: 11 March 2021

Corresponding Author: Alonso Rodríguez-Navarro alonso.rodriguez@upm.es

Handling Editor: Ludo Waltman

Copyright: © 2021 Alonso Rodríguez-Navarro and Ricardo Brito. Published under a Creative Commons Attribution 4.0 International (CC BY 4.0) license.



RESEARCH ARTICLE

Total number of papers and in a single percentile fully describes research impact—Revisiting concepts and applications

Alonso Rodríguez-Navarro^{1,2} and Ricardo Brito²

¹Departamento de Biotecnología-Biología Vegetal, Universidad Politécnica de Madrid, Avenida Puerta de Hierro 2, 28040, Madrid, Spain
²Departamento de Estructura de la Materia, Física Térmica y Electrónica y GISC, Universidad Complutense de Madrid, Plaza de las Ciencias 3, 28040, Madrid, Spain

Keywords: citation analysis, impact indicators, Leiden Ranking

ABSTRACT

This study uses the data provided by the Leiden Ranking 2020 to support the claim that percentile-based indicators are linked by a power law function. A constant calculated from this function, e_p , and the total number of papers fully characterize the percentile distribution of publications. According to this distribution, the probability that a publication from a country or institution is in the global xth percentile can be calculated from a simple equation: $P = e_p^{(2-lgx)}$. By taking the Leiden Ranking PP_{top 10%}/100 as an approximation of the e_p constant, our results demonstrate that other PP_{top x%} indicators can be calculated applying this equation. Consequently, given a PP_{top x%} indicator, all the others are redundant. Even accepting that the total number of papers and a single PP

total number of papers and a single $PP_{top x\%}$ indicator are sufficient to fully characterize the percentile distribution of papers, the results of comparisons between universities and research institutions differ depending on the percentile selected for the comparison. We discuss which $P_{top x\%}$ and $PP_{top x\%}$ indicators are the most convenient for these comparisons to obtain reliable information that can be used in research policy.

1. INTRODUCTION

The rapid progress in the availability of data on research output and faster methods for their analysis are "leading to a quantitative understanding of the genesis of scientific discovery, creativity, and practice and developing tools and policies aimed at accelerating scientific progress" (Fortunato, Bergstrom et al., 2018, p. 1). Among all the analyses that can be done on research output, one of the most important is the efficiency analysis of the research carried out by institutions and countries; this importance is continually increasing in parallel with the increasing importance that research plays in modern economies. Worldwide R&D (research and development) expenditures amounted to \$1,918 billion in 2015 (National Science Board, 2018), and society needs to know the relevance of what research institutions produce with these expenditures and their efficiency in producing it.

Describing this need, 28 years ago Garfield and Welljams-Dorof (1992) began a paper with the following statement: "Government policy-makers, corporate research managers, and

university administrators need valid and reliable S&T indicators for a variety of purposes: for example, to measure the effectiveness of research expenditures, identify areas of strength and excellence, set priorities for strategic planning, monitor performance relative to peers and competitors, and target emerging specialties and new technologies for accelerated development." Since then, and despite this obvious need, a method to measure the effectiveness of research expenditures has not been indisputably established.

1.1. Citation-Based Indicators of Research Performance

Indicators of research performance have been sought for a long time (e.g., Godin, 2003); since Francis Narin (1976) used the term *evaluative bibliometrics*, many indicators have been proposed and those based on citation counts are the most reliable (De Bellis, 2009; Moet, 2005). However, the use of citation counts for scientific assessments has triggered a long-standing debate (Aksnes, Langfeldt, & Wouters, 2019). In the context of this debate, it should be strongly emphasized that citation counts correlate with the scientific relevance or impact of a scientific publication, but they do not always measure the relevance of a specific scientific publication. There are several reasons why many papers receive more or fewer citations than they deserve (MacRoberts & MacRoberts, 1989) or, more commonly, that they receive them belatedly (Garfield, 1980). Even worse, recognition of novelty in science might be delayed and the reporting papers are ignored in short-term citation counting (Wang, Veugelers, & Stephan, 2017). In contrast, when many papers are aggregated the numbers of papers with excessive and scant number of citations are canceled out. In other words: "to a certain extent, the biased are averaged out at aggregated levels" (Aksnes et al., 2019, p. 5). This canceling out cannot be assured with a low number of papers, and this precludes the use of bibliometrics for the evaluation of small numbers of papers, as in the case of individual researchers. It is worth noting that this does not prevent many papers from being correctly evaluated by bibliometric indices; the impediment for their use is that not all papers are correctly evaluated. Unfortunately, this issue is frequently ignored and bibliometric tools are used in the evaluation of researchers (e.g., Kaptay, 2020; Siudem, Zogala-Siudem et al., 2020). In contrast, at the aggregation level of institutions, citation indicators have been validated against peer review (Rodríguez-Navarro & Brito, 2020a; Traag & Waltman, 2019).

As mentioned above, many indicators have been proposed for the research evaluation of institutions and countries, but those based on citation percentiles that refer to worldwide production (Bornmann, 2010; Bornmann, Leydesdorff, & Wang, 2013; Mcallister, Narin, & Corrigan, 1983) have demonstrated superiority and replaced others based on averages (Opthof & Leydesdorff, 2010). Top percentile indicators have been used by the National Science Board of the USA since 2010 (National Science Board, 2010) and by the Leiden Ranking since 2011 (Waltman, Calero-Medina et al., 2012).

Several studies have addressed the need for research performance indicators to be validated against peer review or other external criteria (Harnad, 2009). Many validation studies have been performed, many of them against peer review. In an extensive study testing many indicators, including percentile indicators (HEFCE, 2015), it has been concluded that "results at output-by-author level (Supplementary Report II) [has] shown that individual metrics give significantly different outcomes from the REF peer review process, and therefore cannot provide a like-for-like replacement for REF peer review" (Wilsdon, Allen et al., 2015, p. 138). However, two further studies using the same data have proved that at the university level, which implies a higher aggregation level, top percentile indicators show good correlations with peer review (Rodríguez-Navarro & Brito, 2020a; Traag & Waltman, 2019).

In summary, there is strong evidence supporting the claim that citation-based percentile indicators are excellent tools for the analysis of research outputs. The challenge is to convert these bibliometric indicators into metrics that can be used by "government policy-makers, corporate research managers, and university administrators" (Garfield & Welljams-Dorof, 1992) to calculate the efficiency of research institutions.

1.2. Dichotomous and United Indicators

In a specific discipline and for certain years, a top percentile indicator records the number of papers that an institution has among the set of global papers in that percentile, when they are ranked from the most cited downwards. This evaluation implies the classification of papers published by a research institution in two groups, depending on whether or not they belong to a certain set of global papers. In terms of citations, the two groups are defined depending on whether they are above or below a certain citation threshold—the issue of citation ties has been discussed previously (Schreiber, 2013; Waltman & Schreiber, 2013). This dichotomous classification of papers (Albarrán, Herrero et al., 2017; Bornmann, 2013) leads to the important notion that "dichotomous procedures rely on the idea that only the upper part of the distribution matters" (Albarrán et al., 2017, p. 628). Consequently, in formal terms, dichotomous indicators do not consider papers that are excluded by the criterion. For example, the use of the top 1% or 10% most highly cited papers as a frame of reference (Tijssen, Visser, & van Leeuwen, 2002) implies that the 99% or 90% other papers are not counted. Thus, it seems that the numbers of such papers or of the citations that they received does not matter.

To integrate all papers in the indicators, after counting the papers in percentile ranks, different weights can be assigned to each rank (higher for the ranks with higher citations), and the weighted numbers of papers are added to obtain a united indicator (Bornmann & Mutz, 2011). Leydesdorff and Bornmann (2011) called this type of percentile indicators *integrated impact indicators* because they take into account the size and shape of the distribution, which is very skewed. This approach has been extensively investigated and different percentile ranks and weights have been proposed (Bornmann, 2013; Bornmann, Leydesdorff, & Mutz, 2013; Bornmann, Tekles, & Leydesdorff, 2019; Leydesdorff & Bornmann, 2012; Leydesdorff, Bornmann, & Adams, 2019; Leydesdorff, Bornmann et al., 2011). It is worth noting that weighted counts of publications in ranks do not require that the ranks be based on percentiles (Vinkler, 2011).

The notion of dichotomy, according to which a single top percentile indicator does not take into account the excluded papers, and that a united indicator is needed for research evaluation, would be correct if the numbers of papers in percentiles were unpredictably distributed. But if the numbers of papers in all percentiles obey a function, the number of papers in a single top percentile could be sufficient to determine the numbers in all the other percentiles. This implies that no paper is ignored if only one percentile is used for evaluation, because the number of papers in any percentile is dependent on the function that describes the citation-based distribution of all papers. This type of function occurs frequently in natural sciences. For example, physics textbooks tell us that the pressure (equivalent to percentile) and volume (equivalent to number of papers in the percentile) of gases follow a strict law that depends on the amount of gas (equivalent to the total number of papers) and the temperature (equivalent to the efficiency of the research institution).

A law of this type also exists in bibliometrics. Citations are universally distributed (Radicchi, Fortunato, & Castellano, 2008) and the numbers of papers in top percentiles obey a power law. This power law is a consequence of another basic relationship in citation analysis: the double rank function. "By ranking publications by their number of citations from highest to

lowest, publications from institutions or countries have two ranking numbers: one for the internal and other for world positions; the internal ranking number can be expressed as a function of the world ranking number"; this function is a power law (Rodríguez-Navarro & Brito, 2018a, p. 31). Therefore, by knowing the total number of papers and the number of papers in a single top percentile, the number of papers in any other percentile can be easily calculated. The percentile law can be expressed in the following way:

The probability of publishing a paper in top percentile $x = e_p^{(2-\lg x)}$ (1)

where e_p is a mathematical derivative $(10^{-\alpha})$ of the exponent (α) of the power law that the numbers of papers versus top percentiles obey (Brito & Rodríguez-Navarro, 2018; Rodríguez-Navarro & Brito, 2019). For an institution with the same percentile distribution as the global production, e_p is equal to 0.1 and, in practice, the highest values of e_p are around 0.3.

1.3. Discussion About Size-Independent Indicators

The present study is largely based on Eq. 1. This equation calculates a probability, which is size independent. The usefulness of size-independent bibliometric indicators and of the application of terms such as productivity, performance, and efficiency in research evaluation have been debated (Abramo & D'Angelo, 2016a, 2016b; Glänzel, Thijs, & Debackere, 2016; Ruiz-Castillo, 2016; Waltman, van Eck et al., 2016). That discussion is out of the scope of this study. However, we think that the ideal for a research institution is size independent: that for a given total number of papers, the number of highly cited papers should be as high as possible. This conclusion emphasizes the importance of size-independent indicators for research evaluation purposes, especially the convenience of the e_p constant, because it allows calculation of the probability of publishing a paper at any highly cited level.

Regarding the use of size-dependent and size-independent indicators, there are not many differences. It is worth noting that if we know the cumulative probability function given by Eq. 1, the cumulative frequency of papers in any top percentile is equal to the probability multiplied by the total number of papers. Thus, the most relevant size-dependent indicator of a research system is the total number of publications, because the number of papers in top percentiles is a function of the total number of papers and the e_p constant. Given the exponential nature of Eq. 1 and the range of numerical values between which e_p varies, to produce a significant number of highly cited papers institutions with a low e_p constant must publish many more papers than others that have high e_p constants. In other words, the e_p constant, mathematically equivalent to PP_{top 10%}/100, measures the efficiency of a research system.

1.4. Aims of This Study

The above standpoint indicates that in the research assessment of the publications of institutions or countries, only two parameters—the total number of papers and the e_p constant—are needed to characterize research performance at all citation levels. The former describes the size and the latter describes the efficiency; if both are known, the number of papers in any other top percentile can be calculated. As already described, this notion has theoretical and empirical support (Brito & Rodríguez-Navarro, 2020; Rodríguez-Navarro & Brito, 2018a, 2019), but it has not been tested against a large number of institutions. Therefore, the first aim of this study was to test it at the university level, making use of the detailed information provided by the Leiden Ranking.

The second aim was to investigate which top percentile should be used to compare the research output of different institutions. It is worth noting that when comparing two institutions

by their ratio of publications at different top percentiles, if their e_p constants are different, the ratio will vary depending on which percentile is used for the comparison (e.g., top 10% or top 1%). Even the question of which of the two institutions is ahead and which is lagging might have opposite responses depending on the percentile used for the comparison and their total numbers of publications (see Figure 4 in Rodríguez-Navarro & Brito, 2019).

2. METHODS

For the aims of this study, we took advantage of the detailed data provided by the Leiden Ranking 2020 (https://www.leidenranking.com/; Excel file downloaded on August 21, 2020; these data have been deposited in Zenodo, DOI: 10.5281/zenodo.4603232), using in all cases fractional counting. The Leiden Ranking includes five research fields: "Biomedical and health sciences," "Life and earth sciences," "Mathematical and computer sciences," Physical sciences and engineering," and "Social sciences and humanities." Previous studies in different research fields (Brito & Rodríguez-Navarro, 2020; Rodríguez-Navarro & Brito, 2018a, 2018b, 2020a, 2020b) demonstrate that the calculation of the e_p constant is statistically robust in three of the Leiden Ranking fields: "Biomedical and health sciences," and "Physical sciences and engineering." There is no information in "Mathematical and computer sciences," and in "Social sciences and humanities" the e_p constant has only been studied economics and business (Rodríguez-Navarro & Brito, 2020a).

Therefore, for the purpose of this study, any of the three aforementioned Leiden Ranking fields could be studied. The field of "Biomedical and health sciences" was not the first choice because "health sciences" might be weak in some universities. Between the other two fields, we selected "Physical sciences and engineering" versus "Life and earth sciences." Although the difference is not large, the number of universities with at least four top 1% most cited papers in the Leiden Ranking evaluation periods (4 years) was higher in "Physical sciences and engineering" than in "Life and earth sciences"; this is a comparative advantage, as shown below.

Henceforth, we will keep the notation of the Leiden Ranking: P is the total number of papers and $P_{top x\%}$ is the number of papers in the top x percentile; $PP_{top x\%}$ is the $P_{top x\%}/P$ ratio multiplied by 100.

The Leiden Ranking reports publications for four percentiles (50, 10, 5, and 1) and these are the data that we compared with the calculated data. For the calculation of the number of publications in these percentiles we used Eq. 1, taking the value of the e_p constant as PP_{top 10%}/100. Because of the statistical variability of PP_{top 10%}, the best method for the calculation of the e_p constant is to count the number of papers in 5–10 top percentiles and fit them to a power law (Rodríguez-Navarro & Brito, 2019). However, for the purposes of this study, using the PP_{top 10%}/100 as a substitute for the e_p constant is sufficiently accurate. The same calculation approach was used when we recorded more stringent percentiles, for example 0.02.

Pearson and Spearman correlations were studied using the free statistics software calculators of Wessa (2017a, 2017b). Two-sided *p*-values are always recorded.

3. RESULTS

3.1. PPtop x% Indicators Are Qualitatively Redundant

The numbers of papers in the top percentiles of global publications follow a power law, before and after dividing by the total number of papers (Rodríguez-Navarro & Brito, 2019). By definition, in all universities their PP_{top x%} plots have a common point when the top percentile is 100, and according to Eq. 1, from this point the PP_{top x%} plots diverge if the universities do not have



Figure 1. Double logarithmic plot of the four $PP_{top x\%}$ indicators reported in the Leiden Ranking, $PP_{top 50\%}$, $PP_{top 10\%}$, $PP_{top 5\%}$, and $PP_{top 1\%}$, for three universities that are distant in the ranking. Field of "Physical sciences and engineering," time period 2009–2012.

identical e_p constants. Therefore, if Eq. 1 is correct the order of universities in the Leiden Ranking based on PP_{top x%} should be the same at any of the recorded percentiles: 1, 5, 10, and 50. In practice there will be some deviations, because the number of papers produced by universities is low and the calculation of top percentile data is affected by statistical variability. In fact, the data provided in the Leiden Ranking includes the lower and upper bounds of the stability interval for each university's PP_{top x%} indicator, and overlapping between these bounds in universities is frequent. To avoid this problem, if we select a few universities that publish a high number of papers and that are distant in the ranking, their relative positions will be maintained at all percentiles recorded in the Leiden Ranking. Figure 1 shows that this in fact happens, but this is a small sample, which is not sufficient to demonstrate that Eq. 1 is of general application.

Next, we selected all the universities listed in the Leiden Ranking with more than 2,000 papers in the field of "Physical Sciences and Engineering." This limitation in the number of papers is intended to keep the variability of the PP_{top x%} data as low as possible. Then we calculated the Spearman rank correlation coefficients between the PP_{top x%} data of different percentiles. Table 1 shows the correlation matrix between percentiles in the first (2006–2009) and last (2015–2018) periods recorded in the Leiden Ranking (similar results are found for other periods). The correlation coefficients are high (> 0.9 with a single exception) and the *p* values are very low, from 10^{-33} to 10^{-127} . As might be expected, rank correlations are lower when the top 1% and top 50% results are compared, but are still remarkable. Additionally, Figure 2 shows the least and most dispersed scatter plots of ranks of the correlations studied (Table 1).

These results demonstrate that $PP_{top x\%}$ indicators are redundant, all showing the same ranking information, although their values were obviously very different.

3.2. $PP_{top x\%}$ Indicators Can Be Easily Calculated from $PP_{top 10\%}$

Before addressing the issue of whether empirical $PP_{top x\%}$ indicators follow Eq. 1, for guiding purposes, we addressed a basic descriptive question about the distribution of universities according to these indicators. Figure 3 shows the distributions of universities based on the four

2006–2009	PP _{top 1%}	PP _{top 5%}	PP _{top 10%}	
PP _{top 5%}	0.98			
PP _{top 10%}	0.97	0.99		
PP _{top 50%}	0.94	0.97	0.98	
2015–2018	PP _{top 1%}	PP _{top 5%}	PP _{top 10%}	
PP _{top 5%}	0.96			
PP _{top 10%}	0.95	0.99		
PP _{top 50%}	0.89	0.94	0.96	

Table 1. Spearman rank correlation matrix between the four $PP_{top x\%}$ indicators reported in the Leiden Ranking for universities with more than 2,000 publications

Field of "Physical sciences and engineering." Time periods 2006–2009, 71 universities, and 2015–2018, 151 universities. All 2-sided *p*-values are below 1×10^{-32} .

indicators PP_{top 50%}, PP_{top 10%}, PP_{top 5%}, and PP_{top 1%}, for the time period 2009–2012 (in other time periods the distributions are similar). The PP_{top 50%} distribution resembles a normal distribution and meets normality criteria. The other three distributions show increasing kurtosis, with a long right tail that resembles lognormal distributions. However, although the distributions are heavy tailed, they do not meet the criteria for this type of distribution.

Next, we tested the agreement between the PP_{top x%} indicators reported in the Leiden Ranking and their calculated values from Eq. 1, taking PP_{top 10%}/100 as the value of e_p . In a first attempt we used the data of the 1,177 universities in the field of "Physical sciences and engineering" for the time period 2009–2012. Visually, the scatter plots in Figure 4 show a strong linear relationship between the two values for PP_{top 50%} and PP_{top 5%}. A linear relationship was also observed for PP_{top 1%}, but in this case the data had too much noise. This high variability was due to the large number of universities with a very low number of papers in P_{top 1%}: The value was zero in 203 universities and 1 in 186; in fact, low values of P_{top 1%} are associated with large stability intervals of PP_{top 1%} in the Leiden Ranking. The Pearson correlation coefficients for the



Figure 2. Examples of scatter plots of ranks of the correlations reported in Table 1, the least and most disperse plots. Left panel, $PP_{top 5\%}$ versus $PP_{top 10\%}$ in period 2015–2018, 151 universities; right panel, $PP_{top 1\%}$ versus $PP_{top 50\%}$ in period 2006–2009, 71 universities.



Figure 3. Histograms of the values PP_{top 50%}, PP_{top 10%}, PP_{top 5%}, and PP_{top 1%} reported in the Leiden Ranking for the field of "Physical sciences and engineering" in the time period of 2009–2012; 1,177 universities.

calculated versus the empirical values of PP_{top 50%}, PP_{top 5%}, and PP_{top 1%} were high: 0.89, 0.96, and 0.78, respectively. The *p*-values were very small; the largest was 3.9×10^{-242} for PP_{top 1%}.

Although these correlations were clear, the relationship between the Leiden Ranking and calculated values of $P_{top 1\%}$ was uncertain because the variability could conceal possible deviations of small groups of universities. To overcome this problem the obvious possibility was to exclude from the analysis the universities with $P_{top 1\%}$ values below a certain threshold. This approach, however, had to be carried out avoiding the introduction of biases, which were less likely if the threshold was low. By using the threshold of $P_{top 1\%} \ge 5$, the total set of 1,177 universities was divided into two sets, above and below the threshold, of 474 and 703 universities. The corresponding scatter plots of the Leiden Ranking versus the calculated data of $PP_{top 50\%}$ and $PP_{top 5\%}$ (Figure 5) show high similarity in the two sets and with the scatter plot of the total set of universities (Figure 4). These results suggested that the set of 474 universities was reasonably representative of the total number of universities for the comparison of the Leiden Ranking and calculated values, at least at the $PP_{top 50\%}$ and $PP_{top 5\%}$ levels.



Figure 4. Scatter plots of the two values of PP_{top 50%}, PP_{top 5%}, and PP_{top 1%}, one calculated from PP_{top 10%} and the other the reported in the Leiden Ranking; research field of "Physical sciences and engineering" and time period 2009–2012; 1,177 universities. The lines are meant only to guide the eye.



Figure 5. Scatter plot of the two values of PP_{top 50%} and PP_{top 5%} shown in Figure 4 divided into two sets: $P_{top 1\%} \ge 5$ (a; 474 universities) and $P_{top 1\%} < 5$ (b; 703 universities). The lines are meant only to guide the eye.

For PP_{top 1%}, the set of 703 universities (Figure 6A) shows high variability and the accuracy of fitting a regression line was very low. In the other set of 474 universities (Figure 6B) the variability was lower and the scatter plot reveals that some universities with high values of PP_{top 1%} deviate from the general trend of the other universities. Consequently, a second-order polynomial that passes through the origin fits the data better than a straight line; a higher order polynomial or eliminating the constraint of passing through the origin did not significantly improve the fitting. This finding suggested that a small set of universities could deviate from the relationship of the other universities. It is likely that this possible set of universities might have very large values of either P_{top 1%} or PP_{top 10%}. The scatter plot in Figure 6C shows that the exclusion of 34 universities with P_{top 1%} > 40 does not significantly affect the deviation from a straight regression line observed in Figure 6B. In contrast, the exclusion of 25 universities with PP_{top 10%} ≥ 0.20 eliminates the deviation from a straight regression line. Figure 6D shows that in this case the fittings of straight and polynomial lines overlap.

3.3. Research Efficiency and Contribution to the Progress of Knowledge

Although the total number of papers and their number in a single percentile are sufficient to define the efficiency of research institutions, in the case of quantitative comparisons it is necessary to select the percentile at which the comparison between institutions must be made.



Figure 6. Scatter plot of the two values of PP_{top 1%} shown in Figure 4 divided into two sets: $P_{top 1\%} \ge 5$ (A; 474 universities) and $P_{top 1\%} < 5$ (B; 703 universities). In panels C and D, the set of 474 was subdivided excluding the universities in which $P_{top 1\%} > 40$ (C) and $PP_{top 10\%} \ge 0.20$ (D). Green lines: straight linear regression. Brown lines: fitting to a second-order polynomial. In D, the green and brown lines overlap.

This is so because differences between institutions increase with the stringency of the percentile (Figure 1). However, we must distinguish two different cases, depending on whether we are interested in efficiency, which is size independent, or in the contribution to the progress of knowledge, which is size dependent.

In the first case, if it is necessary to select a PP_{top x%} indicator, the selection might be simple. Considering the data reported in Figure 1 and the exponential form of Eq. 1, it is obvious that the differences increase following a known pattern, which indicates that the ratios between universities' PP_{top x%} also increase or decrease following a known pattern. In these conditions, the convenient percentile cannot be established in general terms and will depend on the target that is pursued (Section 4.2).

If we are interested in the contribution to the progress of knowledge, the relationships between institutions become more complex because, as previously mentioned, the pertinent indicator is the size-dependent $P_{top x\%}$. If the institutions publish similar numbers of papers the case is not different from that described above for efficiency. For example, for Stanford University, Sorbonne University, and Kyushu University in Table 2, the differences increase when the percentile decreases, but the order of the universities does not change. In contrast, if the number of papers is different, even the order of the institutions could change when the stringency of the indicator increases. Table 2 shows this fact again with three universities:

2006–2009							
	Р	P _{top 50%}	P _{top10%}	P _{top 1%}	P _{top 0.1%}	P _{top 0.01%}	
Stanford University	2,825	2,068	741	109	50.97	13.37	
Sorbonne University	2,641	1,518	321	31	4.72	0.57	
Kyushu University	2,669	1,144	188	13	0.93	0.07	
		2009–2	2012				
Shanghai Jiao Tong University	4,832	2,379	437	37	3.57	0.32	
Sorbonne University	2,559	1,483	314	29	4.73	0.58	
Yale University	1,268	916	298	42	16.46	3.87	

Table 2. Variation of $P_{top x\%}$ indicators in selected universities

The values of P, $P_{top 50\%}$, $P_{top 10\%}$ and $P_{top 10\%}$ were taken from the Leiden Ranking, $P_{top 0.1\%}$ and $P_{top 0.01\%}$ were calculated from $PP_{top 10\%}$ as described in the text. Field of "Physical sciences and engineering."

Shanghai Jiao Tong University, Sorbonne University, and Yale University. This is the order (from higher to lower) when using P, $P_{top 50\%}$, and $P_{top 10\%}$, but for $P_{top 1\%}$, Yale University is now first, and the other two universities keep the same order as in the other percentiles. Interestingly, at this percentile the three universities are very similar. Finally, using $P_{top 0.01\%}$, the order changes again: Now, Yale University is first and Sorbonne University is ahead of Shanghai Jiao Tong University. At this percentile, the contribution of Yale University to the progress of knowledge is almost 10 and eight times higher than those of Shanghai Jiao Tong and Sorbonne Universities, respectively. With this complex behavior, the question of which university contributes the most to scientific progress is puzzling, unless we agree about the percentile that should be used to measure scientific progress.

4. DISCUSSION

4.1. All PPtop x% Indicators Can Be Calculated from Only One

The purpose of our study was to demonstrate that Eq. 1 is correct by using the data reported by the Leiden Ranking for a large number of universities. This implies that a single $PP_{top x\%}$ indicator is sufficient to calculate all $PP_{top x\%}$ indicators and therefore to reveal the efficiency of a research institution. The size-independent $PP_{top x\%}$ indicators are 100 times the probabilities described by Eq. 1 and $PP_{top 10\%}$ is equal to the e_p constant multiplied by 100 (Rodríguez-Navarro & Brito, 2019). This constant is normally calculated by statistical fitting from several percentile counts, but it can also be calculated with a lower precision from the value of a single $PP_{top x\%}$.

The high Spearman rank correlation coefficients found between the four Leiden Ranking PP_{top x%} indicators—PP_{top 50%}, PP_{top 10%}, PP_{top 5%}, and PP_{top 1%}—for universities with more than 2,000 papers (Table 1) imply that the four indicators reveal the same as predicted by Eq. 1. The same conclusion is reached when studying the correlation between the PP_{top 50%}, PP_{top 5%}, and PP_{top 1%} data recorded in the Leiden Ranking and the data calculated applying Eq. 1—substituting PP_{top 10%}/100 for the e_p constant. A clear correlation is shown by the three scatter plots for PP_{top 50%}, PP_{top 50%}, PP_{top 5%}, and Pt_{top 1%} in 1,177 universities (time period 2009–2012; Figure 4). However, the scatter plot for PP_{top 1%} is very noisy because in many universities P_{top 1%} is very low and shows a large variability, which hinders the study of deviations that seem to occur. Eliminating the universities with fewer than five papers in P_{top 1%}, there remain 474 universities. Comparison of the scatter plots of the two

sets, 474 and 703 universities, and the complete set of universities (Figures 5 and 6) strongly suggests that the set with 474 universities is a representative sample of the total number of universities and may be used to study possible deviations of $PP_{top 1\%}$.

In Figure 6, the PP_{top 1%} scatter plot shows higher variability than that observed for the PP_{top 50%} and PPtop 5% plots (Figure 5), and the best universities deviate from the trend followed by of the rest of the universities. Several factors contribute to these facts. In the first place, the exponent of Eq. 1 for $PP_{top 1\%}$ is higher than for $PP_{top 50\%}$ and $PP_{top 5\%}$, which increases the error of substituting $PP_{top 10\%}$ for the e_p constant— e_p should be calculated by fitting the data of several percentiles. Furthermore, the number of Ptop 1% papers is low in many universities, which implies a higher variability in the counting of the papers in this percentile than in the counts of the other two percentiles. These general observations are not sufficient to explain the deviations that are observed in Figure 6 for the most efficient universities (panels B, C, and D); we found that by excluding the 25 universities with $PP_{top 10\%} \ge 0.20$ from the set of 474 universities, the deviation from a straight regression line disappears. This result indicates that Eq. 1 suffers slight deviations in highly competitive universities, which would not be surprising, because deviations of empirical data from a general law are common in many scientific fields. In the example of physics given in Section 1.2, the mentioned function applies to ideal gases but suffers deviations in real gases. However, for PP_{top 1%} the deviation is of minor importance for evaluation purposes because the number of these outstanding institutions is an insignificant portion of the total number of institutions: 25 out of 1,177.

In summary, percentile indicators are dichotomous indicators only in appearance, because all of them can be calculated from the total number of papers and a mathematical constant that reveals the research efficiency of institutions and countries. The existence of slight deviations from Eq. 1 in some specific cases does not impede the use of this equation in general evaluations.

4.2. Which Top Percentile Should Be Used for Quantitative Evaluations?

Our data demonstrate that if the purpose is to rank research institutions by the PP_{top x%} indicator, any percentile can be used. Conversely, for quantitative evaluations, such as comparison with research investments (de Marco, 2019), a certain percentile must be selected, because quantitative relationships between institutions change depending on the percentile (Figure 1). For example, let us imagine two research institutions, A and B, in which investments are similar, but the numbers of papers in the evaluation period are 1,000 and 500, and the PP_{top 10%} indicators are 14% and 20%, respectively. It is evident that if we are comparing the cost of a publication, institution A shows the better performance. The same occurs at the top 10% level (P_{top 10%} = P · e_p and e_p = PP_{top 10%}), 140 versus 100 papers, but not at the top 1% level, where both institutions show the same P_{top 1%}, equal to 20 (P_{top 1%} = P · e_p²). At a landmark level (percentile 0.02, Bornmann, Ye, & Ye, 2018) the advantage is for institution B: 0.69 for A versus 1.3 for B (P_{top 0.02%} = P · e_p^{3.7}). Therefore, although A produces twice as many papers than B, the cost of a landmark level paper in A is almost twice the cost as in B.

These bibliometric calculations show the importance of answering the question posed in the title of this section. From a scientific point of view, and if we are considering a size-dependent indicator, the top 0.01 percentile, close to the landmark level, might be a reasonable answer.

For the contribution to the progress of knowledge, the same percentile should apply to scientifically advanced countries and to countries that are developing a research system. Because the target of scientific research is globally established, the research indicator should also be globally established. The same reasoning does not apply to size-independent indicators, because higher is not always better and high-level excellence is not always the right target.

To our knowledge, many research policy makers do not address the evaluative puzzle arising from the example given above, and they choose a certain percentile without much thought. Similarly, in many countries, especially in those with a generally low level of research performance (e.g., Spain), policy makers are preoccupied with the idea of having "excellent" research institutions, and they make important investments in a very few institutions with the purpose of making them "excellent." Aside from the fact that in many cases in these countries research "excellence" is mismeasured by the journal impact factors (Brito & Rodríguez-Navarro, 2019), the results of these efforts are anything but excellent. This is because the contribution to the national research system of an excellent institution will most likely be of low relevance. This would be the case if, for example, in such an institution the PP_{top 10%} is 15% and the average in the rest of the country's institutions is 9.0%, but the "excellent" institutions publish only less than one hundredth of the total number of publications. In this case a simple calculation demonstrates that more than 90% of the top 0.01 publications have been published in the underfunded institutions. Therefore, in countries with weak research systems, investing to raise the average $PP_{top 10\%}$ of the country, for example to 0.12, would be more profitable than investing in the much desired "excellent" institutions.

Another example illustrates why $PP_{top x\%}$ targets have to be adapted to circumstances. In Europe, in the field of technology there are no universities with the $PP_{top 10\%}$ (Leiden Ranking 2020, field of "Physical sciences and engineering") as high as in some U.S. universities, such as Harvard University, Stanford University, and Massachusetts Institute of Technology (MIT). However, at the country level, several European countries have similar or even higher $P_{top 0.01\%}$ per million inhabitants than the United States (Rodríguez-Navarro & Brito, 2018b). In these countries it might be a mistake to pursue universities with the high $PP_{top 10\%}$ of the aforementioned US universities. A country's high $PP_{top 10\%}$ can be obtained from many types of institutions' $PP_{top 10\%}$ distributions, and it seems that each country should pursue the highest possible $P_{top 0.01\%}$ per million inhabitants rather than other targets.

5. CONCLUSIONS

Making use of the data provided by the Leiden Ranking for many universities, we found further empirical evidence supporting the notion that the size-independent $PP_{top x\%}$ indicators are not dichotomous indicators: Any $PP_{top x\%}$ indicator is sufficient to define the research efficiency of a research institution and all $PP_{top x\%}$ indicators can be easily calculated from only one. Therefore, the information given by the Leiden Ranking and the National Science Board of the National Science Foundation, which report several $PP_{top x\%}$ indicators for the same institution or country, is obviously informative, but actually redundant. Similarly, in $P_{top x\%}$ indicators, which are size dependent and measure the contribution of research institutions and countries to the advancement of science, by knowing the total number of papers all $P_{top x\%}$ indicators can be easily calculated from only one, provided that the total number of papers is known.

Both the $P_{top x\%}$ and $PP_{top x\%}$ indicators vary depending on the top percentile selected, which raises the question of which percentile assessments should be made. Our results suggest that for the assessment of contribution to scientific progress, the top 0.01 percentile appears to be the most convenient. In the case of research efficiencies, any single percentile allows comparing countries and research institutions, but for statistical reasons the top 10 percentile might be the best.

The distributions of universities according to $PP_{top x\%}$ indicators ($x \le 10$) are heavy tailed, which implies that the highest probabilities of making important discoveries accumulate in a very low proportion of all universities. Research policy makers should study the $PP_{top x\%}$ indicators of their research institutions before launching research policies that are addressed to the scientific progress of the country.

ACKNOWLEDGMENTS

We thank two anonymous reviewers for their helpful suggestions on improving the original manuscript.

AUTHOR CONTRIBUTIONS

Alonso Rodríguez-Navarro: Conceptualization, Data curation, Formal analysis, Investigation, Supervision, Visualization, Writing—original draft, Writing—review & editing. Ricardo Brito: Conceptualization, Formal analysis, Funding acquisition, Investigation, Methodology, Visualization, Writing—review & editing.

COMPETING INTERESTS

The authors declare that there are no competing interests.

FUNDING INFORMATION

This work was supported by the Spanish Ministerio de Economía y Competitividad, Grant Number FIS2017-83709-R.

DATA AVAILABILITY

The raw data were downloaded from the Leiden Ranking; these data are available at Zenodo (DOI 10.5281/zenodo.4603232).

REFERENCES

- Abramo, G., & D'Angelo, C. A. (2016a). A farewell to the MNCS and like size-independent indicators. *Journal of Informetrics*, *10*, 646–651. **DOI:** https://doi.org/10.1016/j.joi.2016.04.006
- Abramo, G., & D'Angelo, C. A. (2016b). A farewell to the MNCS and like size-independent indicators: Rejoinder. *Journal of Informetrics*, 10, 679–683. **DOI:** https://doi.org/10.1016/j.joi.2016.01.011
- Aksnes, D. W., Langfeldt, L., & Wouters, P. (2019). Citations, citation indicators, and research quality: An overview of basic concepts and theories. SAGE Open, January. DOI: https://doi.org/10.1177 /2158244019829575
- Albarrán, P., Herrero, C., Ruiz-Castillo, J., & Villar, A. (2017). The Herrero-Villar approach to citation impact. *Journal of Informetrics*, *11*, 625–640. **DOI:** https://doi.org/10.1016/j.joi.2017.04.008
- Bornmann, L. (2010). Towards an ideal method of measuring research performance: Some comments to the Opthof and Leydesdorff (2010) paper. *Journal of Informetrics*, *4*, 441–443. **DOI:** https://doi .org/10.1016/j.joi.2010.04.004
- Bornmann, L. (2013). How to analyze percentile citation impact data meaningfully in bibliometrics: The statistical analysis of distributions, percentile rank classes, and top-cited papers. *Journal* of the American Society for Information Science and Technology, 64, 587–595. **DOI:** https://doi.org/10.1002/asi.22792

- Bornmann, L., Leydesdorff, L., & Mutz, R. (2013). The use of percentile rank classes in the analysis of bibliometric data: Opportunities and limits. *Journal of Informetrics*, *7*, 158–165. **DOI:** https://doi.org/10.1016/j.joi.2012.10.001
- Bornmann, L., Leydesdorff, L., & Wang, J. (2013). Which percentilebased appoach should be preferred for calculating normalized citation impact values? An empirical comparison of five approaches including a newly developed citation-rank approach (P100). *Journal of Informetrics*, 7, 933–944. DOI: https://doi.org/10.1016/j.joi.2013.09.003
- Bornmann, L., & Mutz, R. (2011). Further steps towards an ideal method of measuring citation performance: The avoidance of citation (ratio) averages in field-normalization. *Journal of Informetrics*, *5*, 228–230. **DOI:** https://doi.org/10.1016/j.joi.2010.10.009
- Bornmann, L., Tekles, A., & Leydesdorff, L. (2019). How well does 13 perform for impact measurement compared to other bibliometric indicators? The convergent validity of several (field-normalized) indicators. *Scientometrics*, *119*, 1187–1205. **DOI:** https://doi.org /10.1007/s11192-019-03071-6
- Bornmann, L., Ye, A., & Ye, F. (2018). Identifying landmark publications in the long run using field-normalized citation data. *Journal* of Documentation, 74, 278–288. DOI: https://doi.org/10.1108/JD -07-2017-0108

Brito, R., & Rodríguez-Navarro, A. (2018). Research assessment by percentile-based double rank analysis. *Journal of Informetrics*, *12*, 315–329. **DOI:** https://doi.org/10.1016/j.joi.2018.01.011

- Brito, R., & Rodríguez-Navarro, A. (2019). Evaluating research and researchers by the journal impact factor: Is it better than coin flipping? *Journal of Informetrics*, *13*, 314–324. **DOI:** https://doi.org/10.1016/j.joi.2019.01.009
- Brito, R., & Rodríguez-Navarro, A. (2020). The USA dominates world research in basic medicine and biotechnology. *Journal* of Scientometric Research, 9, 154–162. DOI: https://doi.org/10 .5530/jscires.9.2.19
- De Bellis, N. (2009). Bibliometrics and Citation Analysis From the Science Citation Index to Cybermetrics. Lanham, MD: The Scarecrop Press.
- De Marco, A. (2019). Metrics and evaluation of scientific productivity: Would it be useful to normalize the data taking in consideration the investments? *Microbial Cell Factories, 18*, 181. **DOI:** https:// doi.org/10.1186/s12934-019-1236-4, **PMID:** 31655596, **PMCID:** PMC6815394
- Fortunato, S., Bergstrom, C. T., Börner, K., Evans, J. A., Helbing, D., ... Barabási, A.-L. (2018). Science of science. *Science*, *359*, eaao0185. DOI: https://doi.org/10.1126/science.aao0185, PMID: 29496846, PMCID: PMC5949209
- Garfield, E. (1980). Premature discovery or delayed recognition Why? *Current Contents*, *21*, May 26, 5–10.
- Garfield, E., & Welljams-Dorof, A. (1992). Citation data: Their use as quantitative indicators for science and technology evaluation and policy-making. *Science and Public Policy*, *19*, 321–327.
- Glänzel, W., Thijs, B., & Debackere, K. (2016). Productivity, performance, efficiency, impact – What do we measure anyway? Some comments on the paper "A farewell to the MNCS and like size-independent indicators" by Abramo and D'Angelo. *Journal of Informetrics*, *10*, 658–660. **DOI:** https://doi.org/10.1016/j.joi .2016.04.008
- Godin, B. (2003). The emergence of S&T indicators: Why did governments supplement statistics with indicators? *Research Policy*, *32*, 679–691. **DOI:** https://doi.org/10.1016/S0048-7333(02)00032-X
- Harnad, S. (2009). Open access scientometrics and the UK research assessment exercise. *Scientometrics*, *79*, 147–156. **DOI:** https://doi.org/10.1007/s11192-009-0409-z
- HEFCE. (2015). The Metric Tide: Correlation analysis of REF2014 scores and metrics (Supplementary Report II to the independent Review of the Role of Metrics in Research Assessment and Management). **DOI:** https://doi.org/10.13140/RG.2.1.3362.4162
- Kaptay, G. (2020). The k-index is introduced to replace the h-index to evaluate better the scientific excellence of individuals. *Heliyon*, 6(7), e04415. DOI: https://doi.org/10.1016/j.heliyon .2020.e04415, PMID: 32685733, PMCID: PMC7358733
- Leydesdorff, L., & Bornmann, L. (2011). Integrated impact indicators compared with impact factors: An alternative research design with policy implications. *Journal of the American Society for Information Science and Technology*, *62*, 2133–2146. **DOI:** https://doi.org/10.1002/asi.21609
- Leydesdorff, L., & Bornmann, L. (2012). Percentile ranks and the integrated impact indicator (I3). *Journal of the American Society for Information Science and Technology*, 63, 1901–1902. **DOI:** https://doi.org/10.1002/asi.22641
- Leydesdorff, L., Bornmann, L., & Adams, J. (2019). The integrated impact indicator revised (I3): A non-parametric alternative to the journal impact factor. *Scientometrics*, *119*, 1669–1694. DOI: https://doi.org/10.1007/s11192-019-03099-8
- Leydesdorff, L., Bornmann, L., Mutz, R., & Opthof, T. (2011). Turning the tables on citation analysis one more time: Principles for comparing sets of documents. *Journal of the*

American Society for Information Science and Technology, 62, 1370–1381. DOI: https://doi.org/10.1002/asi.21534

- MacRoberts, M. H., & MacRoberts, B. R. (1989). Problems of citation analysis: A crtical review. *Journal of the American Society* for Information Science and Technology, 40, 342–349. DOI: https://doi.org/10.1002/(SICI)1097-4571(198909)40:5<342::AID -ASI7>3.0.CO;2-U
- Mcallister, P. R., Narin, F., & Corrigan, J. G. (1983). Programmatic evaluation and comparison based on standardized citation scores. *IEEE Transactions on Engineering Management*, *EM*-30(4), 205–211. DOI: https://doi.org/10.1109/TEM.1983.6448622
- Moet, H. F. (2005). *Citation analysis in research evaluation*. Berlin: Springer Verlag.
- Narin, F. (1976). Evaluative bibliometrics: The use of publication and citation analysis in the evaluation of scientific activity. Computer Horizon Inc.
- National Science Board. (2010). *Science and engineering indicators*. National Science Foundation.
- National Science Board. (2018). *Science and engineering indicators* 2018. National Science Foundation
- Opthof, T., & Leydesdorff, L. (2010). Caveats for the journal and field normalizations in the CWTS ("Leiden") evaluations of research performance. *Journal of Informetrics*, *4*, 423–430. **DOI:** https://doi.org/10.1016/j.joi.2010.02.003
- Radicchi, F., Fortunato, S., & Castellano, C. (2008). Universality of citation distributions: Toward an objective measure of scientific impact. *Proceedings of the National Academy of Sciences of the USA*, *105*, 17268–17272. DOI: https://doi.org/10.1073/pnas.0806977105, PMID: 18978030, PMCID: PMC2582263
- Rodríguez-Navarro, A., & Brito, R. (2018a). Double rank analysis for research assessment. *Journal of Informetrics*, *12*, 31–41. **DOI:** https://doi.org/10.1016/j.joi.2017.11.004
- Rodríguez-Navarro, A., & Brito, R. (2018b). Technological research in the EU is less efficient than the world average. EU research policy risks Europeans' future. *Journal of Informetrics*, *12*, 718–731. **DOI:** https://doi.org/10.1016/j.joi.2018.06.009
- Rodríguez-Navarro, A., & Brito, R. (2019). Probability and expected frequency of breakthroughs – basis and use of a robust method of research assessment. *Scientometrics*, 119, 213–235. DOI: https:// doi.org/10.1007/s11192-019-03022-1
- Rodríguez-Navarro, A., & Brito, R. (2020a). Like-for-like bibliometric substitutes for peer review: Advantages and limits of indicators calculated from the e_p index. *Research Evaluation*, 29, 215–230. DOI: https://doi.org/10.1093/reseval/rvaa002
- Rodríguez-Navarro, Á., & Brito, R. (2020b). Might Europe one day again be a global scientific powerhouse? Analysis of ERC publications suggests it will not be possible without changes in research policy. *Quantitative Science Studies*, *1*, 872–893. **DOI:** https://doi.org/10.1162/qss_a_00039
- Ruiz-Castillo, J. (2016). Research output indicators are not productivity indicators. *Journal of Informetrics*, 10, 661–663. DOI: https://doi.org/10.1016/j.joi.2016.04.004
- Schreiber, M. (2013). How much do different ways of calculating percentiles influence the derived performance indicators? *Scientometrics*, *97*, 821–829. **DOI:** https://doi.org/10.1007 /s11192-013-0984-x
- Siudem, G., Zogala-Siudem, B., Cena, A., & Gagolewski, M. (2020). Three dimensions of scientific impact. *Proceedings of the National Academy of Sciences USA*, *117*, 13896–13900. DOI: https://doi.org/10.1073/pnas.2001064117, PMID: 32513724, PMCID: PMC7322031
- Tijssen, R. J. W., Visser, M. S., & van Leeuwen, T. N. (2002). Benchmarking international scientific excellence: Are highly cited

research papers an appropriate frame of reference? *Scientometrics*, *54*, 381–397. **DOI:** https://doi.org/10.1023/A:1016082432660

- Traag, V. A., & Waltman, L. (2019). Systematic analysis of agreement between metrics and peer review in the UK REF. *Palgrave Communications*, 5, 29. DOI: https://doi.org/10.1057/s41599-019-0233-x
- Vinkler, P. (2011). Application of the distribution of citations among publications in scientometric evaluation. *Journal of the American Society for Information Science and Technology*, 62, 1963–1928. DOI: https://doi.org/10.1002/asi.21600
 Waltman, L., Calero-Medina, C., Kosten, J., Noyons, E. C. M.,
- Waltman, L., Calero-Medina, C., Kosten, J., Noyons, E. C. M., Tijssen, R. J. W., ... Wouters, P. (2012). The Leiden ranking 2011/2012: Data collection, indicators, and interpretation. *Journal* of the American Society for Information Science and Technology, 63, 2419–2432. DOI: https://doi.org/10.1002/asi.22708
- Waltman, L., & Schreiber, M. (2013). On the calculation of percentilebased bibliometric indicators. *Journal of the American Society for Information Science and Technology*, 64, 372–379. DOI: https:// doi.org/10.1002/asi.22775

- Waltman, L., van Eck, N. J., Visser, M., & Wouters, P. (2016). The elephant in the room: The problems of quantifying productivity in evaluative scientometrics. *Journal of Informetrics*, 10, 671–674. DOI: https://doi.org/10.1016/j.joi.2015.12.008
- Wang, J., Veugelers, R., & Stephan, P. (2017). Bias against novelty in science: A cautionary tale for users of bibliometric indicators. *Research Policy*, 46, 1416–1436. DOI: https://doi.org/10.1016/j .respol.2017.06.006
- Wessa, P. (2017a). Pearson Correlation (v1.0.13) in Free Statistics Software (v1.2.1). Office for Research Development and Education. https://www.wessa.net/rwasp_correlation.wasp/
- Wessa, P. (2017b). Spearman Rank Correlation (v1.0.3) in Free Statistics Software (v1.2.1). Office for Research Development and Education. https://www.wessa.net/rwasp_spearman.wasp/
- Wilsdon, J., Allen, L., Belfiore, E., Campbell, P., Curry, S., ... Johnson, B. (2015). The metric tide: Report of the independent review of the role of metrics in research assessment and management. **DOI:** https://doi.org/10.13140/RG.2.1.4929.1363