Kepler's *Optical Part of Astronomy* (1604): Introducing the Ecliptic Instrument

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The year 2009 marks the 400th anniversary of the publication of one of the most revolutionary scientific texts ever written. In this book, appropriately entitled, Astronomia nova, *Johannes Kepler (1571–1630) developed an astronomical theory which departs fundamentally from the systems of Ptolemy and Copernicus. One of the great innovations of this theory is its dependence on the science of optics. The declared goal of Kepler in his earlier publication,* Paralipomena to Witelo whereby The Optical Part of Astronomy is Treated (Ad Vitellionem Paralipomena, quibus astronomiae pars optica traditvr, *1604), was to solve difficulties and expose illusions astronomers face when conducting astronomical observations with optical instruments. To avoid observational errors that had plagued the antiquated mea-*

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suring techniques for calculating the apparent diameter and angular position of the luminaries, Kepler designed a novel device: the ecliptic instrument. In this paper we seek to shed light on the role optical instruments play in Kepler's scheme: they impose constraints on theory, but at the same time render astronomical knowledge secure. To get a comprehensive grasp of Kepler's astonishing achievements it is required to widen the approach to his writings and study Kepler not only as a mathematico-physical astronomer, but also as a designer of instruments and a practicing observer.

1. Introduction

It is well known that Johannes Kepler (1571–1630) announced in his *New Astronomy* (1609) the discovery that the course of a planet is "an oval path, perfectly elliptical" (Donahue 1992, p. 68; *KGW* 3: p. 35). Kepler reported that he obtained this result by carrying out "most laborious proofs and . . . computations on a very large number of observations" (Donahue 1992, p. 68; *KGW* 3: p. 35). He made it explicit that his success was based on discarding the approach of Ptolemy (*fl.* 150) who had considered the mean motion of the Sun, rather than its apparent motion. According to Kepler, Ptolemy was under the impression that there would be no perceived difference between the two motions, so he chose the former method for ease of calculation (Donahue 1992, pp. 47–51; *KGW* 3: pp. 19–22). Nicolaus Copernicus (1473–1543) and Tycho Brahe (1546–1601) followed Ptolemy, but for Kepler this was not the way to proceed. Already in his early study, *Mysterium cosmographicum* (1596), Kepler indicated that the apparent motion should be taken into consideration, that is, the reference point should be the true body of the Sun (Donahue 1992, p. 121; *KGW* 3: p. 65; Duncan 1999, pp. 159–163; *KGW* 1: pp. 50–54).

But this was not enough. For Kepler a successful matching of calculations with observations is not sufficient; the pattern hidden in the observations has to be accessible to physical explanation. Such a pattern should correspond to the three distinct domains which together constitute for Kepler the framework for the execution of the needed reformation of astronomy: geometry, physics, and observations (Donahue 1992, p. 162; *KGW* 3: p. 93). For a method of inquiry to be successful in this framework it must be in agreement with physical causes (Donahue 1992, p. 49; *KGW* 3: p. 21).

Recent scholarship has been principally engaged with Kepler's attempt at turning astronomy into a physical science (e.g., Goldstein and Hon 2005; Barker and Goldstein 2001; Voelkel 2001; Martens 2000; Aiton et al. 1997, pp. xi–xxxviii; Applebaum 1996; Donahue 1996; Barker and Goldstein 1994; Kozhamthadam 1994; Field 1988; Stephenson 1987; Lindberg 1986; and Rosen 1986). These studies highlight conceptual, theological, metaphysical, epistemological, methodological, and rhetorical aspects of Kepler's *New Astronomy.* Against this rich background we offer a study based on a detailed analysis of what Kepler called the ecliptic instrument, in the hope to raise new fundamental questions concerning the relation between observation and theory in the astronomical context. We contend that understanding Kepler's astronomical achievements takes more than his archetypal principles, and concerns for Aristotelian philosophy, Neo-Platonism, mathematics, mechanics, as well as his own insights concerning a new synthesis of natural philosophy and mathematics. Kepler's engagement with astronomical observations is complex and we seek to contribute towards its clarification.

Kepler noted that astronomers measure with instruments the distances between the fixed stars, planets, and even the edges of the Sun and Moon, and express these measurements in arcs of visual angles *(anguli visorii).* These arcs are based on geometrical divisions of angles and arithmetical calculations—they are in effect constructs of the mind. To put it bluntly, arcs and their divisions do not exist in the physical world. The astronomical enterprise, therefore, has to rest upon optical reasoning, the only way to guarantee a reliable link between a mental construct and the physical reality of heavenly bodies (Donahue 2000, p. 321; *KGW* 2: pp. 267–268).

Optics and astronomy are interwoven. Kepler writes,

> Because of the eccentric, the planets appear either slow or fast. The cause is partly physical, partly optical. The physical part of the cause does not give the sense of vision a reason for error, but also represents to the vision that which in fact occurs, [an account] of which is in the *Commentaries on the motions of Mars.* (Donahue 2000, p. 339; *KGW* 2: p. 282)

Indeed, Kepler presented in the *New Astronomy* the physical causes of the motions of the heavenly bodies. But, according to Kepler, of no less importance is the study of the optical causes related to astronomical observations. Kepler presented the study of these causes in his *Paralipomena.* Visual illusions and poor optical reasoning plagued the observations taken by Kepler's predecessors. According to Kepler these astronomers believed "in the theorem without restrictions"[1] and thus "fell into a large error" (Donahue 2000, p. 57; *KGW* 2: p. 48) in estimating the motions of the planets in relation to their courses and distances from Earth. But for Kepler

1. By "the theorem" Kepler meant the method then used to measure with a compass "the magnitudes [apparent diameters] of solar eclipse, the ratios of the [apparent] diameters of the sun and moon, and the inclinations to the vertical of the circle drawn through the centers of the luminaries" (Donahue 2000, p. 57; *KGW* 2: p. 48).

> the error was immediately obvious from the apparent magnitude [of the sun], for the epicycles increased less at perigee than accords with such a close approach, for which reason another cause for the slowing down was seized upon, which, as I have just said, Ptolemy ascribed to the circle of the equant. In the sun, no epicycle was needed, and as a consequence this error has remained to this day. It was, however, first discovered by me, through an exact observation of the visible diameter, as I shall say below [in the *Paralipomena*], and then by Tycho's most precise observations taken of the star Mars, as I shall make plain at the proper time and place [in the *New Astronomy*]. (Donahue 2000, p. 341; *KGW* 2: pp. 283–284)

As early as 1600, while considering the relation between theory and observation, Kepler remarked that "for these speculations a priori must not conflict with clear experimental evidence, indeed they must be in conformity with it."[2] The need to match the calculations of each planet's path with the observed positions of the planet at different places on its course led Kepler to examine the optical part of astronomy. In 1602 Kepler expressed his opinion about the relation between optics and the study of the motion of the planets:

> I have committed myself to accomplish two goals: The first, to be completed by Passover is the commentary (or whatever its name will be) on the theory of Mars, or the key to a universal astronomy, dealing with the problems of second motions, as a result of Tycho Brahe observations. The second, the optical part of astronomy, is to be completed within 8 weeks, and will be of great importance, for what you [Herwart] encouraged me [the investigation of the motion of the planets]. In this work, from my observations which are the foundation of computations and hypotheses and from many pieces of information, I collect something general for the consideration of one who wishes to contemplate a theory of luminous bodies. (Kepler To Herwart, November 12, 1602; *KGW* 14: pp. 299–300)

In the event it took seven more arduous years to accomplish the first goal, namely, the theory of Mars, and two years to complete the second objective: the optical part of astronomy. It is noteworthy that as early as 1602 Kepler had known that he would be able to conceive a new astronomical theory, and it is equally striking that he first opted to complete his optical studies—a fundamental aspect of his new astronomy.

2. Kepler to Herwart, July 12, 1600; *KGW* 14: p. 130: "non debent enim hae a priori speculations in manifestam impingere experientiam: sed cum hac conciliarj."

Kepler realized that "supposing the place of the celestial body to be known with complete precision, throws the demonstrations into difficulty: the nature of light, beset by the inconstancy of optical causes, does not always allow such precision of instruments" (Donahue 2000, p. 6; *KGW* 2: p. 8). Observations have their limits and what is required is to bring the calculations within a valid margin of accuracy. In Kepler's words: "by the best reasoning at our disposal, we have brought the calculation within the limits of observable error" (Donahue 1992, p. 559; *KGW* 3: p. 355; see also Hon 2004, pp. 72–79 and Hon 1987). It is evident that in Kepler's view no idea concerning astronomy was of any use unless it was supported by observations (Kepler To Fabricius, July 4, 1603; *KGW* 14: p. 412), which in turn could be carried out only with instruments. Kepler emphasized time and again the need to study how astronomical instruments function so that one could control the way observations and measurements are made, and determine how reliable they are (Donahue 2000, pp. 13, 15–16, 56–57, 157–158, 171, 227, 231–232, 350–351; *KGW* 2: pp. 14, 15–16, 47–48, 134–135, 143–144, 190, 193, 290–291). This demand is most prominent in Kepler's analysis of the pinhole camera.

In this paper we revisit Kepler's study of the pinhole camera. We discuss his theoretical insights and their application in turning the pinhole camera into an astronomical instrument, as presented in chapter 11 of Kepler's *Paralipomena.* This optical study is, among other things, a *tour de force* of geometrical analysis of working instruments. This is, so to speak, an applied geometry that takes into consideration the material limitations of instruments. We seek to bring together historical and technical perspectives on some of Kepler's optical procedures and to examine the results he obtained from the application of an ingenious arrangement of two different instruments. In his ecliptic instrument Kepler combined the pinhole camera (for aiming purposes and measuring the apparent diameters of the luminaries) with the quadrant (for executing positional astronomy). We follow Kepler identifying problems, seeking solutions, developing an experimental setup to test his argument, and finally putting forward an optical theory. This close study of Kepler's theoretical insights into a novel optical apparatus, the ecliptic instrument, sheds light on the role instruments play in imposing constraints on epistemology, but at the same time making knowledge claims secure.

2. The problem

In the 16th century the pinhole camera was at the center of interest of scholars from different disciplines. The application of this device as an optical imaging instrument for astronomical measurements posed an intriguing problem for astronomers, mathematicians, and opticians *(magis opticus).*[3] One such scholar was Tycho who in the latter part of the century modified the pinhole sights of his astronomical instruments. He found that the most suitable arrangement of pinholes is that in which the lower sight, the one closest to the eye, has slits on all four sides.[4] Tycho fitted this sight with a graduated disk which he then used as a screen on which the upper pinhole cast a circular image when pointed toward the Sun (Thoren 1973, p. 29). This improvement allowed aiming the instrument accurately and the observer could also measure the Sun's apparent diameter by the comparison of the projected image with the disk.

In March 1578, Tycho tried a different method: to calculate the apparent solar diameter from measurements of the width of the Sun's image cast on the floor through a slit *(rima)* in the wall (Donahue 2000, pp. 353–354; *KGW* 2: pp. 293–294; Straker 1981, pp. 272–274; Straker 1970, pp. 332–334). In his solar observation of 1591 Tycho used yet another method to calculate the apparent diameter of the Sun. He furnished a wooden quadrilateral tube *(canalis)* with a square pinhole through which the Sun's image was projected onto a screen placed relatively close to the pinhole. Tycho's measurements of the cast images of the Sun and the Moon involved large errors (Donahue 2000, pp. 352–353; *KGW* 2: p. 292; Straker 1970, pp. 336–345, 354–356, 399–406). Following his observations of solar eclipses in the year 1598, Tycho reported in a letter to Michael Mästlin (1550–1631) at Tübingen,

> The moon during a solar eclipse does not appear to be the same size as it appears at other times during full moons when it is equally far away; but it appears as if it were constricted by about 1/5th. . . . As a result, it appears that the moon can never obscure the sun completely, and even if the moon interposes itself centrally, the remaining light of the sun encircles it. . . . Although the diameter of the moon . . . by our calculations ought to have been 34¾′, it could not have appeared in front of the sun to be more than 28′, which constriction I recognized and was noticed by no one before me. (quoted by Straker 1981, p. 278; see also Donahue 2000, p. 57; *KGW* 2: p. 48)

3. For historical accounts and discussions on light and theories of pinhole images in the Latin West, see Hon and Zik 2007; Ilardi 2007, pp. 219–224; Lefèvre 2007; Thro 1996; Thro 1994; Mancha 1989; Goldstein 1985; Lindberg and Cantor 1985; Lindberg 1984; Straker 1981; Lindberg 1976, pp. 185–188; Beer and Beer 1975, pp. 789–861; Lindberg 1970; Straker 1970; Lindberg 1968.

4. See the appendix, *Supplementum de subdivisione et dioptris instrumentorum:* Brahe 1598, appendix [82]. For an account of Tycho's astronomical instruments and the operational limits of his new sighting arrangement, see Thoren 1973 and Wesley 1979, p. 99.

Tycho appealed to a purely observational reasoning. His empirically based method of observation was developed without an adequate theory of image formation behind small pinholes (Straker 1981, pp. 267, 276; Donahue 2000, pp. 57, 352–354; *KGW* 2: pp. 48, 292–294). He noticed a lunar diminution during solar eclipses and, therefore, entered in his lunar tables smaller values for the apparent diameter of the Moon than he had actually measured (Straker 1981, p. 278). Astronomers at the time were familiar with Tycho's data which were inconsistent with the observations they had made (Straker 1981, pp. 280, 283; Donahue 2000, pp. 297–298; *KGW* 2: p. 249). Kepler, for example, considered the results Tycho had obtained an indication that the Moon may be actually farther away than lunar theory predicts. Something must have been wrong either in astronomy or with Tycho's observations (Starker 1981, p. 278).

The traditional view maintained that the Sun's apparent diameter does not vary perceptibly from apogee to perigee, and is seen under the same angle. Mathematicians and astronomers of the time thought that the Moon's apparent diameter is only seen to be equal to the Sun's diameter when the full Moon is situated at the apogee of its epicycle, while at perigee the Moon appears to be greater than the Sun (Donahue 2000, pp. 57, 309–310; *KGW* 2: pp. 48, 257–258). They, however, were confused by the variety of methods and differences in the results of measurements of the apparent diameters of the Sun and Moon (Donahue 2000, pp. 310–311; *KGW* 2: pp. 257–258). Kepler too was intrigued by the observational fact "whether the ray be allowed in through a notch or received by the eyes, always show the moon's diameter to be much less than it appeared at oppositions" (Donahue 2000, p. 297; *KGW* 2: p. 248). In chapter 5 of his *Paralipomena,* Kepler had already explained "the causes, from the actual structure of the sense of sight, why the edges of luminous objects are [seen] enlarged, particularly in darkness" (Donahue 2000, p. 298; *KGW* 2: p. 248). This aspect of the functioning of the eye affects the way the apparent diameter of the Moon is perceived in the dim light of the eclipsed Sun and Kepler, therefore, proceeded to study phenomena related to the cast shadow of the Moon and daytime darkness during solar eclipse (Donahue 2000, p. 298; *KGW* 2: p. 248). A similar phenomenon of this nature occurs when a pillar obstructs the view of the whole Moon, while a bright white halo is still distinguished from the Moon's body. Kepler sought to simulate this phenomenon; he writes,

> In the evening, the moon being near perigee, . . . I set up a bronze wheel [figure 1], precisely circular, clasped with a spike to one end of a pole twelve feet long [3657.6 mm], applied the eye at the other end, and before the eye a very narrow opening in another

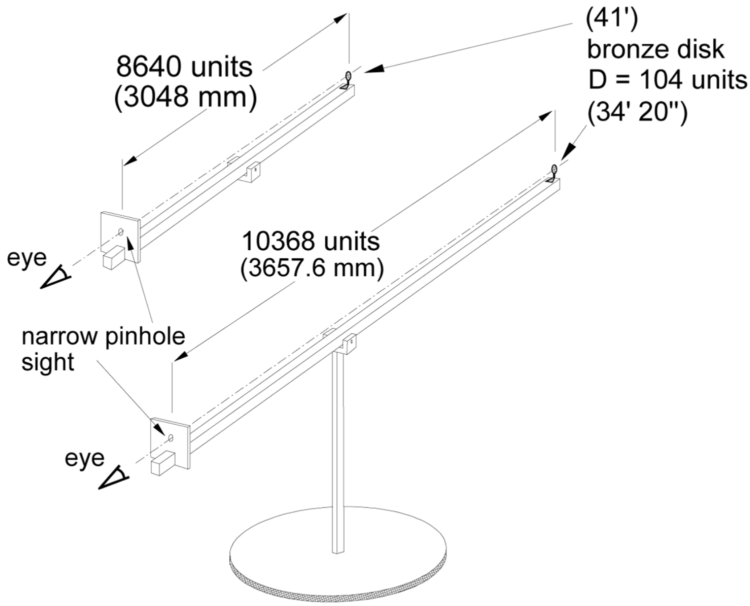

Figure 1. Observing the Moon obstructed by a bronze wheel

> bronze sheet, so that the eye would have a perpendicular view to the wheel through the opening. . . . And since, where the distance between the eye and the wheel was 10368 [units], the width of the wheel would be 104, covering an arc of $34\frac{1}{3}$ minutes, I was hopeful that the moon was going to be completely covered by this wheel, because of other ways of observing, which I was applying at the same time. And in fact, the moon was seen to protrude all around. Here I was troubled by anxiety that perhaps other ways of observing, in which I had the greatest trust, might be false. But that there was a fallacy became immediately plain when the eye was brought nearer the wheel. For all that brightness still did not betake itself behind the wheel even when the eye came to a distance of 10 feet. In this way the moon would have had to represent more than 41 minutes, which every one knows is false. I was also unable to recognize a determinate distance from which the moon would be covered, for I always saw something bright on the circumference. (Donahue 2000, pp. 311–312; *KGW* 2: pp. 259–260)

Kepler noticed the considerable difficulties in the prediction of eclipses and in the reconstruction of the Moon's motion (Donahue 2000, p. 57;

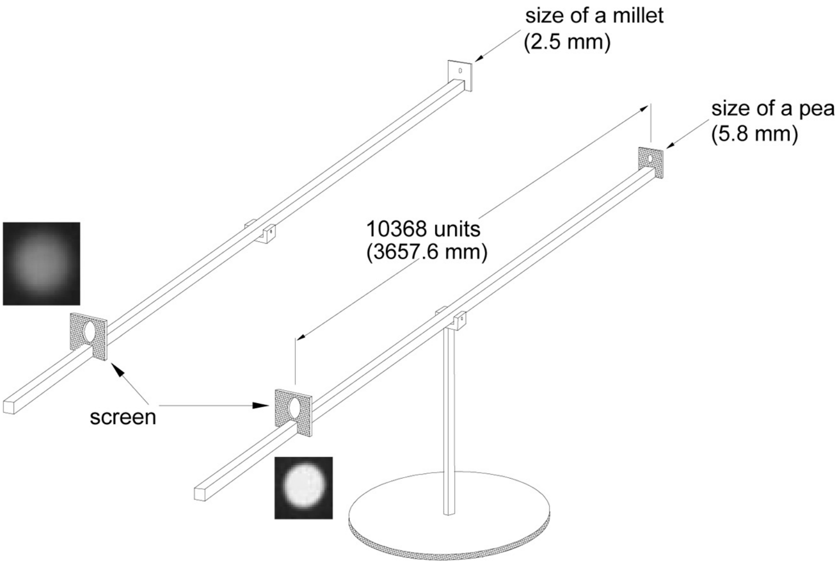

Figure 2. Projection of the sun through pinholes of different diameters

KGW 2: p. 48). As indicated, he attributed the problems to the methods then used to measure the apparent diameters of solar eclipse and the ratios of the apparent diameters of the Sun and Moon. Since the measurements of the apparent diameters of the luminaries had been taken through small holes, Kepler explored how the image of the Sun is cast behind holes of different diameters,

> When two holes were opened [figure 2], one the size of millet, the other the size of a pea, and a pair of circles were painted on the opposite wall [screen], one of which exceeded the other by an interval that was as great as the difference between the larger and smaller holes, the ray of the sun admitted through the larger hole was indeed equal to the greater circle, but when the larger hole was blocked, the ray that came through the smaller did not maintain an evident boundary, and had the edge gradually passing over into a dusky color, and finally, far exceeded the smaller circle. For the ray of the sun, greatly weakened through such a small hole, was unable to illuminate the paper much more brightly than the rays from the air standing about the sun whose continuation with the solar rays portrayed a breadth greater than the truth, and a brilliant color (Donahue 2000, p. 312; *KGW* 2: p. 260).

Kepler pondered whether the experiments with the bronze wheel and the two different pinholes could simulate the astronomical conjunction when the Sun is obscured by the full Moon. His first experiment confirmed that the full Moon indeed appears to the observer larger than its real size, but then the second experiment resulted in more questions than answers. Though Kepler described his experiments of the projection of the Sun's images through two circular pinholes of different diameters, his account suggests that he had actually experimented with a variety of pinhole set-ups. Kepler, however, concluded that, since the sense of sight varies by individual cases, the projected images of the Sun and Moon are seen differently by different people (Donahue 2000, pp. 297–298; *KGW* 2: p. 248). These differences are illustrated in figure 3 where projections and multiplication of the Sun's image by different shapes and diameters of pinholes as well as their distances from the screen are presented.

Kepler warned astronomers that they must not trust the sense of sight when they measure the apparent diameter of the full Moon and that of the Sun (Donahue 2000, p. 298; *KGW* 2: p. 249). In contrast to Tycho, Kepler was convinced that the wide variations in the power of sight of individual observers make it impossible to establish correction tables. He remarked,

> it cannot . . . be argued from this accident of the sense of sight to what happens outside of consideration of the sense of sight, nor can tables be established for the sake of the sense of sight, which represent neither the object itself nor the defects of all senses of sight. For the astronomer should not present anything other than those things that in actual fact occur. The sense of vision, however, we leave to the physicians to remedy (Donahue 2000, p. 298; *KGW* 2: p. 249).

Kepler concluded that the luminous boundaries of the eclipsed Sun's image, projected through a slit or a pinhole, spread out considerably. Hence, the problem: the projected image simply does not show the true diameter of the luminaries and the observer is easily deceived. Kepler, therefore, endeavored first to gain a better understanding of the optical causes of this phenomenon and then to "teach how to enter into a most certain procedure for measuring the quantities of eclipses" (Donahue 2000, p. 298; *KGW* 2: p. 249).

3. Image formation behind a pinhole: some modern optical considerations

Kepler worked within the tradition of geometrical optics. This tradition was restricted to situations where—in current optical terminology—the size of the pinhole is large compared to the wavelength of the light

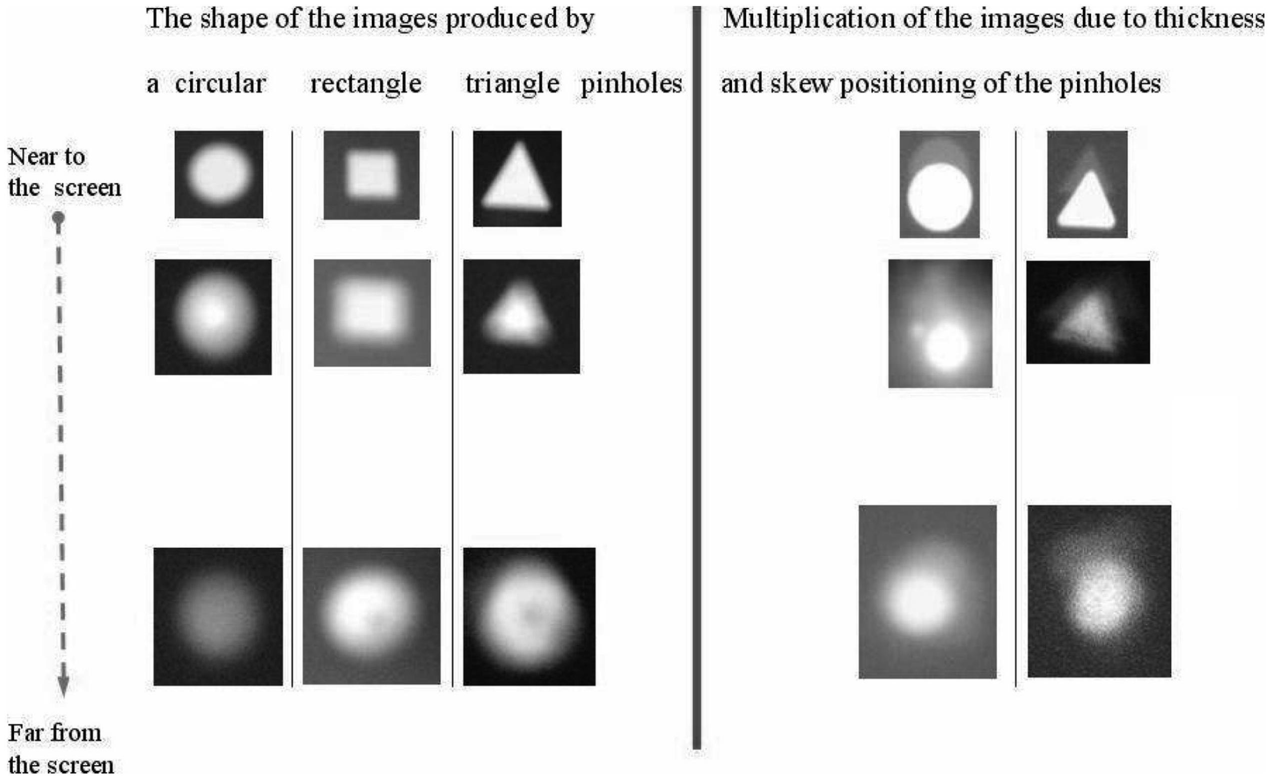

Figure 3. Projected images from different shapes and diameters of pinholes as well as different distances from a screen

transmitted through it or the distance from the pinhole to the screen is short compared to the width of the pinhole.

To have a better understanding of the argument which Kepler developed, it is instructive to recall in modern terms how an image is formed behind a pinhole. Let the rays issued from the center of the Sun, C, (figure 4) and a peripheral point, A, be projected through a square pinhole ED onto a screen, where the centers of the cast images, GF and KH, are denoted by, C′ and C″, respectively. The shift between the images (C′C″) depends on the radius of the apparent-mean Sun ($a = \sim 16$ arc minutes) and the distance, L, between the pinhole and the screen which is calculated trigonometrically as, $C'C''=L \tan a$. Each point source of the Sun creates a rectangular image of the pinhole on the screen and the overlapping partial images produce a different composite image when the screen is placed near to or far from the pinhole. In the latter case the radius of the composite image is the sum of $C'C''+C''K$; that is, the radius of the Sun's image plus the half size of the square pinhole side (in the case of a circular pinhole, $C''K$ would be its radius).

For example, the shift between the images projected on a screen placed at a distance of 200 mm from the pinhole would be 0.93 mm. Thus, for a square pinhole of 5.8 mm (figure 4.1), the shift is much smaller than the partial images created by different point sources on the Sun which practically coincide and produce an image of the square pinhole whose edges are therefore slightly blurred. In a different situation (figure 4.2), the shift between the partial images, projected through the same square pinhole on a screen placed at 3657 mm from the pinhole, would be 17 mm. Since the shift of 17 mm is larger than the size of the partial images, the composite image, whose radius is 19.9 mm, takes the shape of the Sun whose circumference is slightly blurred.

Image formation behind a pinhole is part of the complex optical phenomenon of diffraction which occurs when obstructions (such as pinholes) modify the amplitude, or phase, of a portion of a wave front that encounters these opaque obstacles. Light does not travel in straight lines; it behaves like wave, bending around corners and obstructions, albeit to a finite small degree. Diffraction phenomena are the result of interaction of a large number of light waves whose mathematical analysis is extremely intricate. Some of the phenomena Kepler referred to are caused by what we call today near-field diffraction (Huygens-Fresnel diffraction) and far-field diffraction (Fraunhofer diffraction).[5]

5. For an account of interference and diffraction, see Hecht 1990, pp. 333–339, 392–471. On the diffraction effects of apertures, vignetting, and resolution of optical systems, see Smith 1990, pp. 135–139, 148–155.

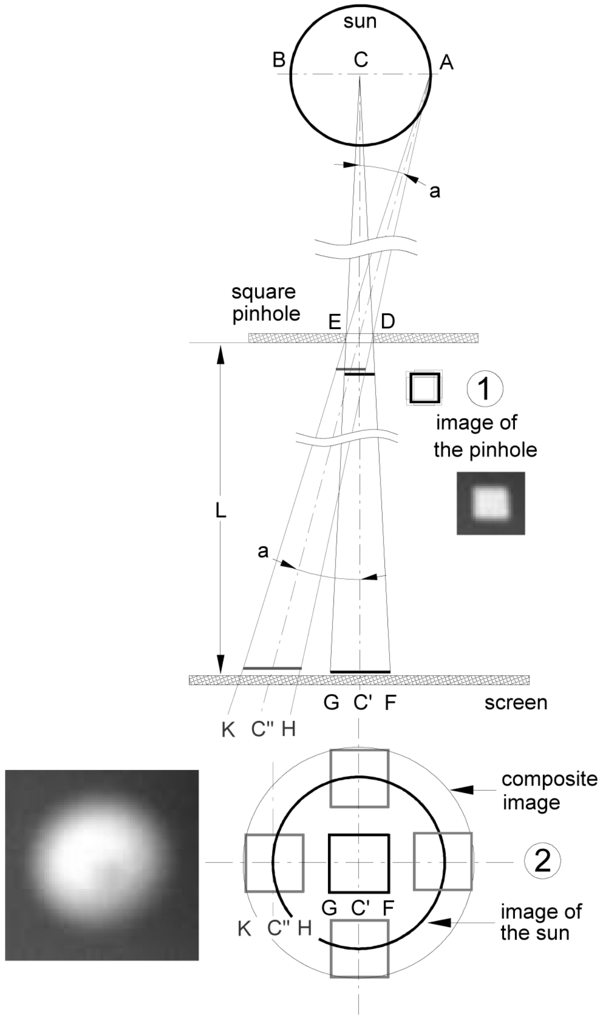

Figure 4. Geometrical analysis of image formation behind a squared pinhole.

Geometrically, rays issued onto a screen through a point pinhole that are perfectly free from any aberration would converge from an axial object-point onto one and the same image-point. Thus, for whatever size of the pinhole and for any distance from the screen, the projected image invariably would be sharp and take the shape of the pinhole. But these considerations are idealized; in reality, a faithful depiction occurs only when the screen is placed very close to the pinhole. No matter what pinhole system

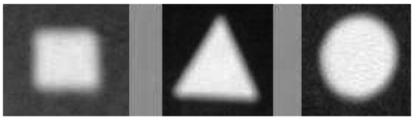

Figure 5. Indistinct edges of the image due to the appearance of light and dark fringes.

Figure 6. The diffraction fringes bordering a peg clasping a coin.

we use, we will always find that when the size of the pinhole is gradually reduced, or the screen is moved away, the edges of the image become indistinct due to the appearance of light and dark fringes (figure 5).

The effect is illustrated in figure 6: a bright pattern caused by the energy-density distribution borders the edge of the shadow cast by a peg clasping a coin. The same happens when light passes through a large pinhole; the bright bordering region diffuses the light in the image.

Circular pinholes produce patterns similar to those shown in figure 6, except that the energy-density distribution exhibits radial symmetry. When the pinhole is reduced, or the screen moved beyond a critical distance, the shape of the pinhole is no longer discernible and the image becomes larger instead of smaller. Therefore, an extended light source projects the exact geometrical shape of the pinhole itself only when the pinhole is placed very close to the screen. Even when the edge is smooth to a very high degree, one observes near-field diffraction patterns, no matter how large the pinhole is. When the distance between the screen and the pinhole grows (figure 7) the pattern of the projection becomes more structured and the multiplications of the image are more prominent.

When the screen is moved farther away, far-field diffraction (Fraunhofer diffraction) effects arise. The projected shape of the pinhole becomes blurred and, as the distance increases, the blurred image bears increasingly a resemblance to the shape of the light source (figure 8). Farther away, the shape of the light source will have spread out considerably, changing only

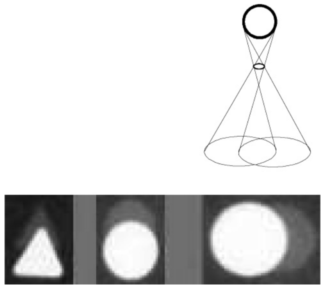

Figure 7. The multiplication of the image.

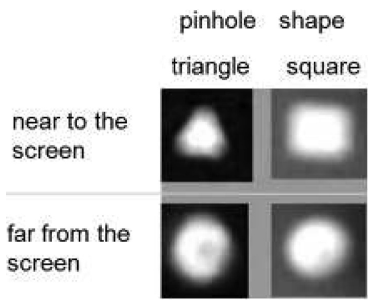

Figure 8. As the distance between the pinhole and the screen grows, the shape of the image increasingly resembles the shape of the light source.

the size of the pattern and not its shape. The boundary between near-field and far-field diffraction depends upon the relative sizes of the pinhole and its distance from the screen. Since far-field domain may be calculated only approximately, the exact location of this boundary is problematic. As a rule of thumb the boundary between near-field and far-field diffraction can be expressed in terms of Fresnel number which is defined:

$$F_{\text{(Fresnel number)}} = r^2 / (L\,\lambda),$$

where r is the radius of the pinhole, L is the distance of the pinhole from the screen, and λ the wavelength involved. Depending on the Fresnel number F, diffraction theory may be associated with two distinct domains: (1) near-field diffraction in which the Fresnel number is $F \geq 1$, and (2) far-field diffraction in which the Fresnel number is $F < 1$. Thus, the Fresnel number, which is an expression of the initial conditions, determines the domain of validity of diffraction theory. The intermediary boundary between the two domains marks the distance *L* that produces the best resolution of pinhole imaging system (Young 1989). For example, for a circular pinhole of 5.8 mm in diameter at wavelength of 5500Å, the boundary between near-field and far-field diffraction would be at a screen distance of about 15.3 meter from the pinhole.

The pinhole camera had to be constructed and operated with care. When the hole is not evenly bored, or not perfectly round (i.e., with ragged edges), or when the thickness of the material in which the pinhole is drilled is relatively large with respect to the diameter of the hole, the cast diffused patterns will blur the image considerably (figures 9.1). Another set of problems may arise due to skew positioning or when the pinhole and the screen are not parallel (figure 9.2 and 9.3).

These forms of stray light, spread out through the pinhole system, obscure faint signals, decrease the signal to noise ratio, reduce contrast and, in general, disturb accurate projection of the image, resulting in poor optical performance.

We position now a pinhole system, pointing directly to the Sun (figure 10). The light passes through the pinhole, bored in a plate, projects an image of the Sun onto a screen placed at distance L. Geometrically, only part *Da* of the image is exposed directly to the light of the Sun across the whole width of the pinhole.[6] Cut off by the sides of the pinhole (at points a, b, c, d), image *Di* is partially illuminated by the light of the Sun in addition to illumination resulted from edge diffraction and light scattering from the inner sides of the pinhole. Though the illumination at the margins of *Di* decreases, the faint edges of the image are still visible. The diameter of the image *Di,* for a given width of the pinhole, is reduced due to the thickness of the pinhole. When projected through a thin pinhole, however, the margins towards the periphery of the image (the circumference between *Di* and *Da*), will be smaller in comparison to the margins projected through a thicker pinhole.

Optical processes can be recast as superposition of light waves.[7] Phenomena like polarization, interference, and diffraction share a common conceptual basis since they deal with various aspects of superposition. The image formed behind the pinhole is considered—in modern terms—a

6. Our analysis concerns the working of pinholes in the domain of near-field diffraction (Huygens-Fresnel diffraction).

7. Superposition occurs when two or more light waves overlap in some region. The resultant form of the composite disturbance is determined by the specific properties of each constituent wave, that is, by its amplitude, phase, frequency and other physical parameters (Hecht 1990, p. 242).

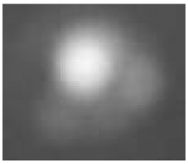

Figure 9.1. Diffused patterns cast by light rays due to thickness and ragged edges of the pinhole.

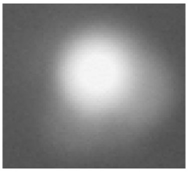

Figure 9.2. Diffused patterns cast by light rays due to skew positioning.

Figure 9.3. Pattern cast by light rays when the pinhole and screen are not parallel.

superposition of an infinite number of images which are slightly offset against each other. Thus, the projected image is the integral of all the irradiance (i.e., the flow of energy per unit of area per unit of time) of the object and the distribution function of illumination caused by impulse response of the optical system (Hecht 1990, pp. 242–269, 472–515). Superposition is a process which results in blurring the input image and, in the case of pinhole projection, also affects the magnitude of the image. Thus, the diameter of an image projected by a pinhole system working at the domain of near-field diffraction will be smaller than it should be in comparison to the true apparent diameter of the Sun.

An analysis of the total irradiance map for absorbed flux of the Sun, projected through a pinhole onto a screen, demonstrates the amount by

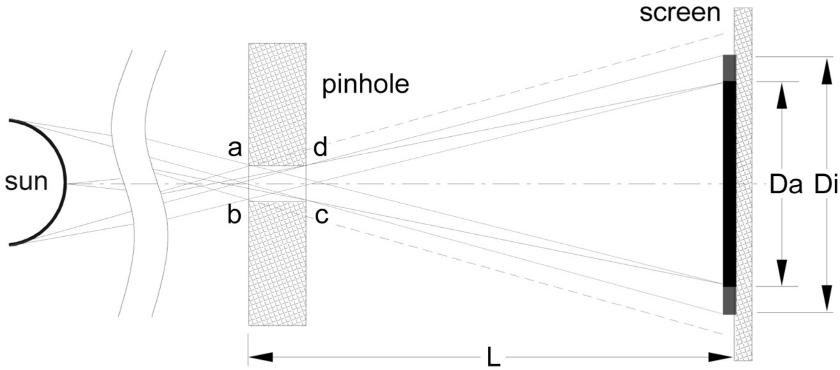

Figure 10. Geometrical analysis of the rays passing through the pinhole

which the image is diminished.[8] As shown in figure 11, the diameter of the image, *Di,* presenting the total irradiance map of the Sun, is composed of different levels of intensity. The irradiance at the center is substantially higher than the irradiance distribution towards the margins of the image. There is a considerable drop-off in the intensity of irradiance at a distance denoted by the diameter, *Da.* The fuzzy demarcation line between diameter, *Di,* and diameter, *Da,* measured on the irradiance map is a good approximation for the diameter of the pinhole. We simulate projection through a pinhole system when the Sun presents an apparent angle of 31′ 50″.[9] In this case the expected apparent diameter of the total irradiance map should be the sum of the apparent diameter of the Sun (17.88 mm) plus the diameter of the pinhole (3 mm); that is, 20.88 mm. However, as shown in the irradiance map, the diameter of the projected image is 20.04 mm. The map suggests a diminution of about 4% in the diameter of the Sun's image. The same analysis is made when the Sun presents an apparent angle of 32′ 30″.[10] In this case the expected apparent diameter of the total

8. Flux density, measured in Watts/m^2, is the quantity of light flow. The irradiance map was made by ELOP using TracePro. This is a powerful modeling software for analyzing and simulating various system-performance criteria and constraints, including spatial and angular light-output distribution, uniformity, intensity, and spectral characteristics. TracePro offers scientists and engineers the confidence that the performance of the finished product will agree with the simulated design without costly prototype iterations.

9. The properties of the pinhole in these simulations are: diameter, 3 mm; thickness, 0.1 mm and 1.5 mm; and the distance of the screen from the pinhole, 1931 mm.

10. The properties of the pinhole are in these simulations: diameter, 5.8 mm; thickness, 0.1 mm and 1.5 mm; and the distance of the screen from the pinhole, 3658 mm.

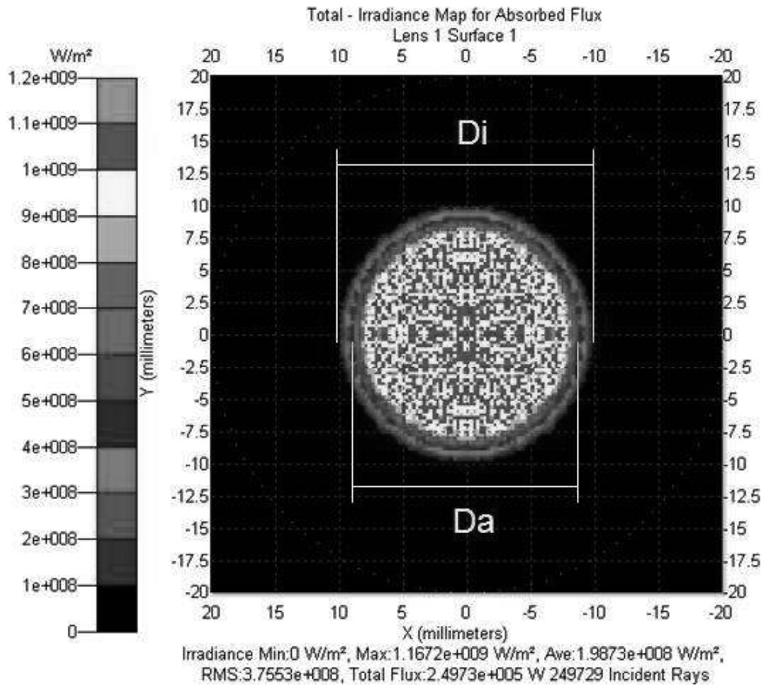

Figure 11. *Di* denotes the diameter of the Sun's total irradiance map depicted on the screen. The demarcation line between diameter, *Di,* and diameter, *Da,* is a good approximation for the diameter of the pinhole.

irradiance map should be the sum of the apparent diameter of the Sun (34.58 mm) plus the diameter of the pinhole (5.8 mm), that is, 40.38 mm. However, as measured in the irradiance map the diameter of the projected image is 38.8 mm. The map suggests, once again, a diminution of about 4% in the diameter of the Sun's image.

All in all, cut-off effects caused by the sides of the pinhole camera, stray light, and the complex process of superposition, contribute to the diminution of an image projected by a pinhole system working at the domain of near-field diffraction. Our calculation shows that it is about 4% of the apparent diameter of the Sun. For example, taking the apparent diameter of the Sun at perigee as 32′ 30″, the projected image will be a diameter of about 31′ 12″. For another example, the apparent diameter at apogee is 31′ 30″, while the image will be a diameter of about 30′ 14″. With this optical analysis in mind, we now follow Kepler's path in which he first developed a theory of the projection of light through pinholes and then applied it in the design and construction of an accurate astronomical instrument, by far better than the instruments employed by Tycho.

4. Kepler and the pinhole camera

To analyze the problem of image formation behind a pinhole, Kepler adopted the thread model that Leon Battista Alberti (1404–1459) and Albrecht Dürer (1471–1528) had used in their perspective instruments *(instrumento).*[11] Kepler reached a geometrical solution by appealing to an experiment which helped him elucidate the highly obscure descriptions provided by his predecessors:

> I [Kepler] set a book in a high place, which was to stand for a luminous body. Between this and the pavement a tablet with a polygonal hole was set up. Next [figure 12], a thread was sent down from one corner of the book through the hole to the pavement, falling upon the pavement in such a way as to graze the edges of the hole, the image of which I traced with chalk. In this way a figure was created upon the pavement similar to the hole.
>
> The same thing occurred when an additional thread was added from the second [figure 13], third, and fourth corners of the book, as well as from the infinite points of the edges. In this way, a narrow row of infinite figures of the hole outlined the large quadrangular figure of the book on the pavement. It was thus obvious that this was in agreement with the demonstration of the problem, that the round shape is not that of the visual ray but of the sun itself, not because this is the most perfect shape, but because this is generally the shape of a luminous body. (Donahue 2000, p. 56; *KGW* 2: p. 47)

Kepler established the geometrical relations between the object and its image by physically following with a thread the path of a ray. He realized that there were more variables to account for. Kepler writes (figure 14),

> If an window [O] could be a mathematical point [*punctum mathematicum*], the illumination of the squarely interposed wall [*FGH*] would precisely assume the shape of the illuminating surface [*NPQ*], but inverted; and the ratio of the diameters of the luminous surface and the illuminated wall would come out the same

11. For Alberti, see Spencer 1966, pp. 68–69; Dürer 1532, pp. 183–185; Lindberg 1976, p. 186; Straker 1970, p. 391. Daniello Barbaro (1514–1570) describes an instrument used by Dürer as an aid for drawing perspective (1569, p. 191).

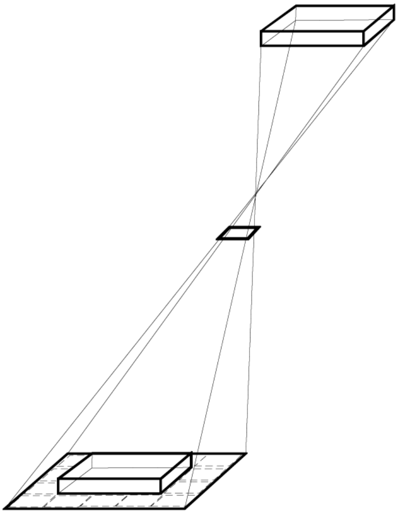

Figure 12. A figure, similar to the shape of the hole, is created on the pavement

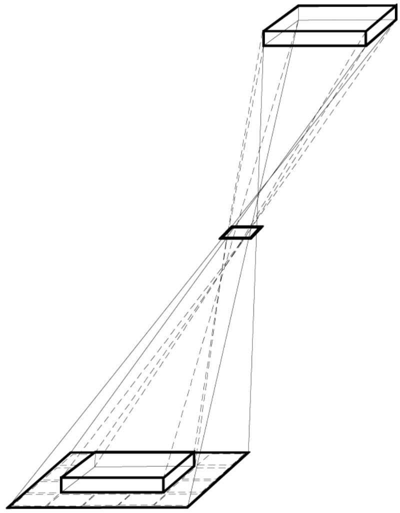

Figure 13. Infinite numbers of figures of the hole outlining the figure of the book on the pavement

> as that of the distance of each from the point of the window. (Donahue 2000, p. 60; *KGW* 2: p. 51)

Whereas ancient and medieval natural philosophers had explained the formation of pinhole image in terms of a double visual pyramid and a point pinhole, Kepler dealt with the phenomenon in terms of pencils of rays emanating from point sources on the object. He considered the finite pinhole a collection of point-holes, which together projected inverted images of the shape of the luminous source. The pinhole diameter and the distance at which the screen was to be placed were of great significance for Kepler's argument (Donahue 2000, pp. 68–69; *KGW* 2: pp. 58–59). He first established the geometrical basis for this analysis.

> Let there be the straight line *NEP* [figure 15.1], divided into equal parts at *E,* and let *FOG* be equidistant[12] from it, likewise divided into equal parts at *O.* Let the straight lines *PO, EF* be drawn until they meet at *K.* In the same way, let *EO* and *NF* or *PG* be drawn until they meet at *I,* and let *K* and *I* be joined. I say that *KI* is equi-

12. Donahue translates the Latin term, *aequidistare* (and its cognates), as "equidistant." In scholastic mathematics *aequidistare* connotes "being parallel." Hence, in this passage, all four instances of "equidistant" stand for "parallel."

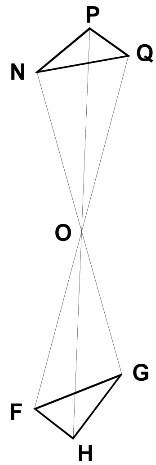

Figure 14. The geometrical relations between the object and its image

distant from the bases *NEP* and *FOG.* For in the triangles *NEI, FOI,* the angles *NEI, FOI* are equal, by Euclid I 29. The same is true of *ENI, OFI,* and the angle at *I* is common. Therefore, the triangles have equal angles, and by Euclid VI 4 the sides are proportional. Therefore, as *NE* is to *FO* so is *EI* to *OI.* In the same way it is proved in *EPK* and *FOK* that as *EP* is to *FO,* so is *PK* to *OK.* But *EP, EN* are equal. Therefore, as *NE* is to *FO,* so is *PK* to *OK.* But previously *EI* was also to *OI* in the same ratio. As a result, as *PK* is to *OK* so is *EI* to *OI.* And by Euclid V 5 as *PO* is to *OK,* so is *EO* to *OI,* and alternately, as *PO* is to *OE* so is *KO* to *OI.* Further, *EOP* is equal to its vertical angle *IOK.* Therefore, by Euclid VI 6, the triangle *EOP, IOK* have equal angles, and *OKI* or *PKI* is equal to *OPE* or *KPE.* As a result, by Euclid I 28, *EP* and *KI* are equidistant. The same is also true of *EPK* and *EPI,* which in contrast have a common base but cut off equal equidistant portions of *FG.* The proposition is therefore evident. (Donahue 2000, pp. 63–64; *KGW* 2: p. 54)

Kepler then proceeded to determine the point where the window should be placed with respect to the luminous surface and the screen,

Let *NEP* be diameter of the luminous surface {figure 15.2}, and *FOG* be the diameter of the window {pinhole}, equidistant from it and lying perpendicularly below it, and let *EO* be the perpendicular to the two. From *N* and *P* let straight lines be drawn through the ends *F* and *G,* until they meet. Let the point of meeting be *I.* I say

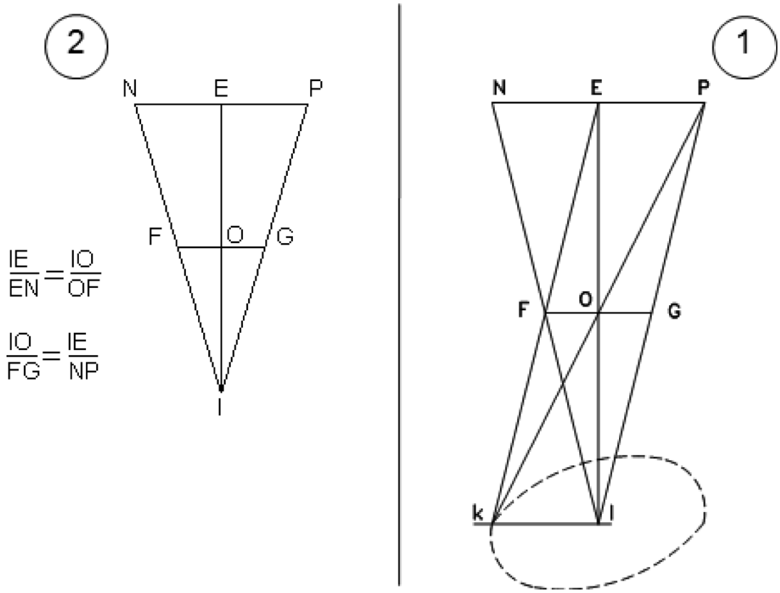

Figure 15. Image formation as a function of the distance between the pinhole and the screen

> that this is the required point. For since in triangle *NIE, FO* is equidistant from the side *NE, IE* will be to *EN* as *IO* is to *OF,* with the result that as *IO* (the distance of *I* from *O*) is to *FG* (twice *OF,* which is the diameter of the window), so is *IE* (the distance of *I* from the luminous surface *E*) to *NP* (twice *NE* and the diameter of the luminous surface), which was to be accomplished. (Donahue 2000, p. 64; *KGW* 2: pp. 54–55)

The ratios EI/EN = OI/OF, and OI/FG = EI/NP (figure 15.2) facilitate Kepler's insight that the diameter of the pinhole must be less than the apparent diameter of the luminous surface.

Kepler concluded that since the rays intersect at points K, L of surface I (figure 16.2), KL is the common measure of both images, namely, the inverted luminous object and the pinhole. As the screen is brought closer to the pinhole, to point X (figure 16.1), the part MT represents the diameter of the inverted image of the luminous object, and HV is the diameter of the image of the pinhole. HV is greater than MT; it is the image of the shape of the pinhole O. Then (figure 16.3), when the screen is moved farther away from the pinhole, to point Y, the rays will intersect at points A,

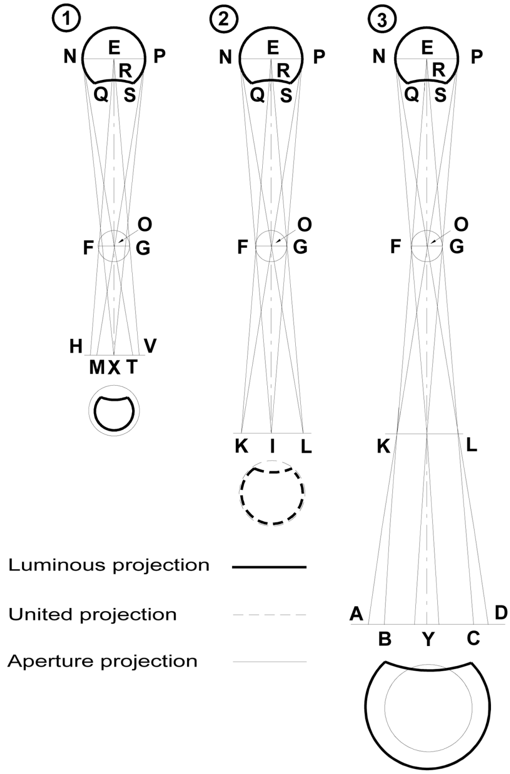

Figure 16. The three domains of diffraction: (1) near-field, (2) intermediary, and (3) far-field. The domain is determined by the distance of the screen from the pinhole

B, C, D. Since the intersection of the rays already occurred at K and L (figure 16.2), the rays that were on the inside are now on the outside at points A, D. The image AD is the inverted image of the luminous object and is greater than the diameter of the image BC that the pinhole projects on the screen. AD takes the shape of the luminous object (Donahue 2000, pp. 65–67; *KGW* 2: pp. 55–57).

It is instructive to recast Kepler's analysis in current optical terminology. We identify three domains, depending on the distance between the pinhole and the screen. Figure 16.2 presents a plane, KL, on which the cast images of the Sun and that of the pinhole coincide. As shown earlier, this plane for a given pinhole diameter (5.8 mm) and wavelength (5500Å), would be at a distance corresponding to the Fresnel number 1;

that is, KL should be placed about 15.3 meter from the pinhole. This distance results in the best resolution for a pinhole imaging system, but the arrangement yields a dim, almost imperceptible image. Moreover, the large distance is not practical and therefore not conducive to astronomical usage. Figure 16.3 shows a projection under the conditions of far-field diffraction. This arrangement is also impracticable because the image is faint and the distance of the pinhole from the screen is too large for manipulation. Figure 16.1 presents an image under the conditions of near-field diffraction. The radius of the composite image HV is the sum of the radius of the Sun's image XM, plus the radius of the pinhole MH. The trade-off between the relatively short distance of the screen from the pinhole and a large diameter of the pinhole yields discernable bright contrast of the cast images so that accurate measurements could be taken.

How is the apparent diameter of the Sun calculated from the measured image? Let AK (figure 17.1) be the radius of the projected image on the screen, AB the radius of the pinhole, and KC, ID, EG, are equal to radius AB. As shown in figure 17.1, the radius of the projected image AK includes both the image of the pinhole as well as the image of the Sun. The radius of the combined image, AK, and the radius of the image of the Sun itself, AC, is shown in figure 17.1 and 17.2 respectively. Kepler then explains how to calculate the apparent diameter of the Sun: When the radius, AB, or CK, is subtracted from the radius of the image, AK, the remainder will be, AC. The radius of the apparent Sun, AC, measured on the screen, is then divided by the distance of the pinhole from the screen. Kepler calculates the tangent of half the apparent angle of the Sun from which he obtains the apparent diameter of the Sun (Donahue 2000, p. 351; *KGW* 2: p. 291).

During an eclipse (figure 17.3), the image of the Sun is covered, fully or partially, by the image of the Moon. In order to measure the apparent diameter of the eclipsed Sun one needs, according to Kepler, to draw a circle from the center of the image of the Moon (figure 17.1), L, through points H, B, and F. Then, add the radius of the pinhole, BA, to the radius of the Moon, LB. Draw from the center of the Moon a circle D, A, C, with the radius LA, representing the true body of the Moon. In the same way, subtract from AG the radius of the pinhole, GE. Draw from the center A circle D, E, C, with the radius, AE, representing the body of the Sun. Kepler claimed that DECA is the image of the eclipsed Sun (Donahue 2000, p. 364; *KGW* 2: p. 302).

Kepler put forward a reliable and accurate measurement procedure for the investigation of the apparent sizes and distances of the Sun and Moon. He established a geometrical framework within which he could analyze the formation of pinhole images and execute two crucial calculations:

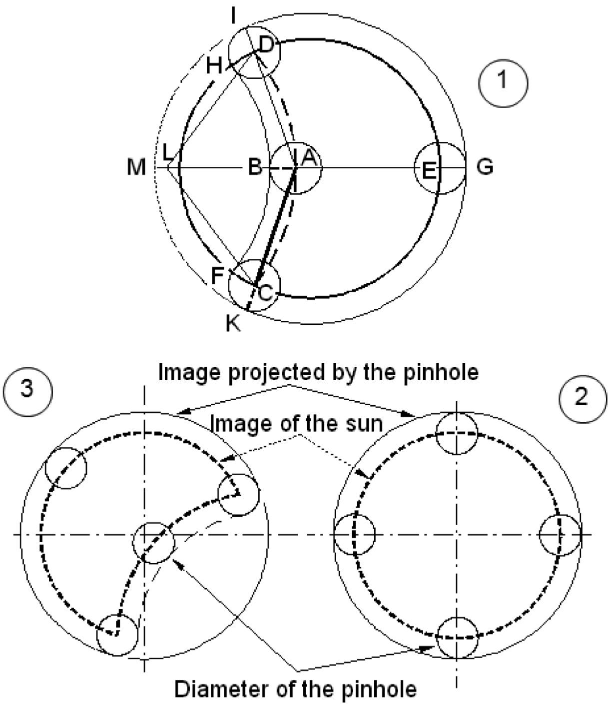

Figure 17. Kepler's method for calculating the apparent diameter of the Sun and Moon

(1) the apparent diameter of the Sun, and (2) the apparent diameter of the Sun when eclipsed. Kepler sought a way to apply his new insights to improve the accuracy of his novel instrument—the ecliptic instrument.

5. Kepler's ecliptic instrument

As part of his reexamination of past claims, Kepler analyzed the astronomical instruments of his predecessors and determined their limitations.[13] It was not enough for Kepler just to know how a projected image is cast be-

13. Among the scholars Kepler mentioned are: Hipparchus (190–120 BC), Ptolemy, Proclus (410–485), Gemma Frisius (1508–1555), Federico Commandino (1509–1575), Erasmus Reinhold (1511–1553), Giambattista Della Porta (1535–1615), Mästlin, and Tycho. Kepler was familiar with Tycho's book (1598): *Astronomiae Instauratae Mechanica* (Donahue 2000, p. 232; *KGW* 2: p. 194). According to Kepler, Ptolemy's observations were never taken with accuracy better than 10 arc minutes, compared to the much more accurate measurements of Tycho, accurate up to 3 arc minutes (Donahue 1992, pp. 201 and 286; *KGW* 3: pp. 120 and 177).

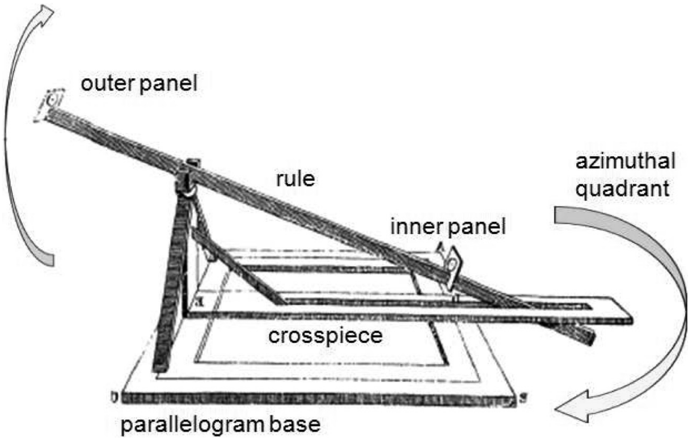

Figure 18. Kepler's ecliptic instrument (notice the horizontal positioning of the crosspiece and the vertical possible orientation of the rule, as well as the panels on the rule)

hind a pinhole; he realized the need to comprehend processes of deception of sight, the working of the eye, and methods of observation. In his *Paralipomena,* chapter 5, Kepler analyzed the deception of sight associated with instruments and methods of observation. He further examined the optical structure of the eye and the geometrical principles of sight. He remarked, for example, that without exception observers report that bright images appear greater in proportion than those that are less bright; this and other perceptual considerations had to be accounted for (Donahue 2000, pp. 171–236; *KGW* 2: pp. 143–197). He noted the importance of technique and skill: proper aiming and alignment, firm support, clear marking of the edges of the image, as well as accurately dividing the graduation scale (Donahue 2000, pp. 16, 56–57, 67–69, 157–158, 224–226, 229–236, 310–311, 319, 341, 352–353, 358–359, 362–363).

To avoid observational errors involved in old techniques, Kepler designed an instrument for measuring the Sun's angular position and apparent diameter. In chapter 11 of his *Paralipomena* Kepler introduces an ecliptic instrument that consists of a solid wooden base (figure 18), which formed a right angled parallelogram in the place of the azimuthal circle. The parallelogram supports a long rotating crosspiece beam with a slit in the middle. The crosspiece holds in its slit an adjustable rule in the vertical plane. The rule is set in such a way that it can be rotated as well as raised towards the zenith or lowered towards the horizon, as much as the altitude of the Sun at the beginning and the end of an eclipse requires.

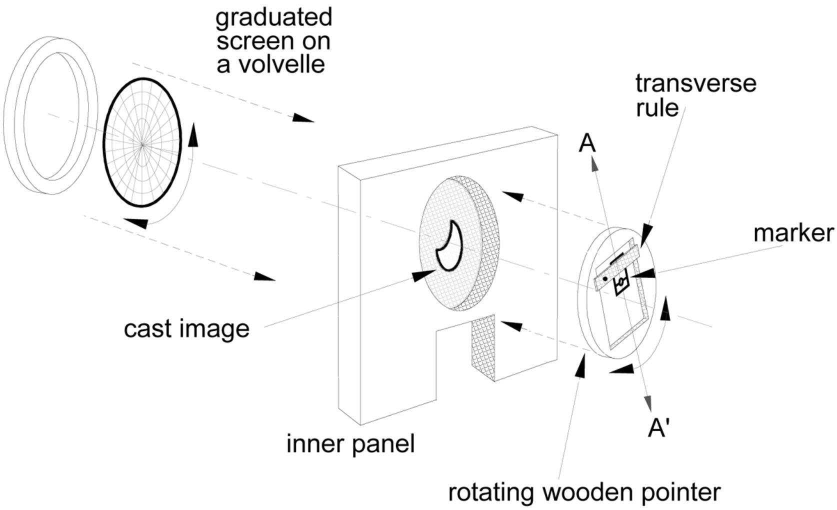

Figure 19. Application of the volvelle and the rotating pointer

This construction is in effect a quadrant for positional astronomy. Kepler, however, did not furnish his ecliptic instrument with transverse graduated scales. Instead of measuring degrees, he rather preferred to measure length on three different linear scales: the azimutal quadrant, the crosspiece, and the rule. On the basis of these measurements Kepler carried out a series of trigonometric calculations in order to determine the position of the luminary in question (Donahue 2000, pp. 348, 381–382, 424–429; *KGW* 2: pp. 290, 319, 357–363; see also *KGW* 21, 1: p. 244).

The rule carries two perpendicular panels parallel to each other. The outer panel has a pinhole while the inner panel is a movable screen on whose upward facing surface a graduated circle, as great as the width of the panel allows, is inscribed (figure 19). The center of the panel is equipped with a volvelle mounted with a rotating disk where the outline of the image is drawn and measured.[14] The space between the panels must be covered with a tube which is black on the inside, so that no light may enter except through the pinhole in the front panel. Kepler gives an account of the physical properties of the instrument he used in the context of discussing the observations made in June and July 1600 and December 1601. The distance between the panels was 12 feet, divided into 10368

14. A volvelle is a medieval device consisting of concentric rotating graduated disks, used to compute the phases of the Moon and its position in relation to the Sun (Donahue 2000, pp. 347–350; *KGW* 2: pp. 288–289).

units, and the diameter of the pinhole, perforated in a very thin sheet of bronze, was 16.5 units (Donahue 2000, pp. 350, 352, 367, 369; *KGW* 2: pp. 290, 292, 304–305, 306–307).[15] For every screen distance there is then an optimal pinhole diameter, which produces the sharpest possible image.

Because of the difficulty in marking points and drawing the cast image on the graduated screen (figure 19), Kepler attached to the volvelle a special pointer made of solid wooden parts in such a way that it did not cover the diameter of its graduated circle divided into digits. Above the solid pointer he placed a transverse rule which could rotate perpendicularly to the pointer and to the marked diameter. Kepler made a groove in the transverse rule equal in its depth to the thickness of the wood of the pointer, and attached the transverse rule to the wooden piece so that one could move it back and forth along the groove. When the instrument is oriented to the Sun, the pointer with the volvelle could be rotated on its axle, and the transverse rule moved along the pointer, until the sharp edge of a marker, placed on the transverse rule, comes into contact with both marginal edges of the image. Kepler marks points where the sharp edge of the transverse rule, placed on the pointer, cuts the diameter of the image. The locations where the interior circumference of the image, or the Moon's shadow, cuts the same diameter are also marked, so Kepler could draw accurately the circumference of the image of the eclipsed Sun (Donahue 2000, pp. 364–365; *KGW* 2: pp. 302–303).

The points inscribed on a paper attached to the screen, sometimes marked with a grid, facilitate the drawing of the circumference of the cast images and to calculate the apparent diameters for both the Sun or the Moon. Kepler describes how to obtain the true image of the eclipsed Sun:

> Let CDEF [figure 20] be the image of eclipsed ray, whose center is G, the marked diameter DF, and with it, the line of pointer IDH [line AA′ in figure 19]. And let there be the line CHE [as shown in figure 20, CHE corresponds to the transverse rule on which the marker is placed] touching the obtuse horns C, E, and cutting the marked diameter at H; and let the segment CFE cut the same diameter at F. Next, let the radius of the opening be extended on the diameter DH, and let this be DL, from D to L, as well as from F to M and from H to P, and through P let the straight line NPO be perpendicular to the diameter and equidistant from the rule. Then, with center G and radius GL let the circle NLO be drawn, which will represent the true image of the sun, as in 9 of this chapter.

15. The length of the rule equals to 3658 mm and the diameter of the pinhole is about 5.8 mm.

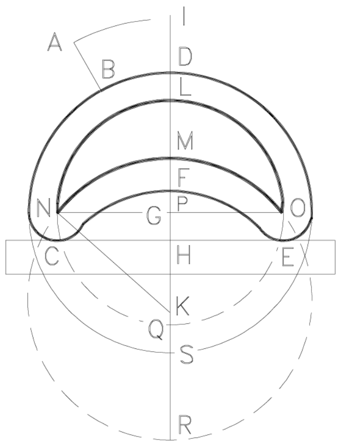

Figure 20. Drawing the circumference of the eclipsed Sun

> This will cut the line NPO: let it cut at N, O. Next, through the three points N, M, O, let a circle be described, and let its center be K. (Donahue 2000, pp. 365–366; *KGW* 2: pp. 303–304)

Kepler concludes that the circumference, NLOM, delineates the true image of the eclipsed Sun. He writes,

> Let the perpendicular NC, OE be drawn [figure 20]. So, since NC, OE are perpendicular to the same CE, they will be equidistant. And because DK, that is, PH, is also perpendicular to the same CE, from the structure of the instrument, therefore, NC, PH, OE will be equal. But PH is the radius of the opening. Therefore, NC and OE are also. But since, by 10 of Chapter 2, the small circle of the opening is also drawn around the points of the ends of the horns, and since CHE is tangent to those small circles (for they are the obtuse horns of the ray), it is necessary that NO be those points of the end of the horns. For from any point of the circumference NLO above N or O, a longer line would be dropped, and from others below N, O, a shorter line, than NC, OE, the radius of the hole in the opening. Therefore, if N, O, are the points of the horns, the circle of the moon cuts the solar circle there. But M is also on the moon's circle, as in 9. Therefore, NMO is the moon's circle. And NLO is the sun's. Therefore, the true image is NLOM. (Donahue 2000, p. 366; *KGW* 2: p. 304)

Kepler's method for measuring the apparent diameter of both the Sun and the Moon is formulated in the following argument (figure 20). The image of the eclipsed Sun, projected onto a screen through a point-hole *(punctum mathematicum),* casts a figure, NLOM; since the shape of the pinhole is also cast onto the screen, the image of the Sun is increased by the radius of the pinhole, DL, to form the composite figure, CDEF. While the projected image of the Sun is increased, the image of the Moon, on the screen (NMO), is decreased by the same amount: the radius of the pinhole, that is, MF. There arises, therefore, the need to take account of the size of the pinhole and to make the appropriate correction when calculating the apparent diameters of the Sun and the Moon during a solar eclipse.

To maximize the performance of the ecliptic instrument, in terms of contrast and brightness of the cast image, the diameter of the pinhole needs to be adjusted. On the one hand, if the pinhole were to be too small, the eyes dazzled by the brightness of daylight would have to be held in place for a long time before they may see the faint image projected in an abundance of light. On the other hand, if the pinhole were to be enlarged the image would be much brighter and more distinct than with a smaller pinhole, but at the same time it would be proportionally coarser and blurred. Kepler opted for an optimal solution: the pinhole should be of an intermediate size. He instructed further that it is helpful to place a cover, like a brow, on the outside, above the upper pinhole to keep "the sky or the air" (that is, stray light) from falling on the screen (where the image is depicted) with bright light (Donahue 2000, pp. 68–69; *KGW* 2: pp. 58–59). In order to minimize observational errors, Kepler also emphasized the need to repeat the measurements of the diameter of the image carefully,

> because unsteadiness will hinder you, with half of what you took with the compass [*circino*], from the center of the panel (which takes the place of the wall), draw a circle, and one slightly smaller than that, and again one slightly larger, as many times as you think necessary. Then investigate again which of the described circles the ray [image] equals. (Donahue 2000, p. 351; *KGW* 2: p. 291)

The radiation of a luminous object is a right cone whose vertex is at the pinhole. The screen must be positioned perpendicular to the source of radiation, so that the projected image would be of a circular shape and not skewed (Donahue 2000, pp. 67–69, 362; *KGW* 2: pp. 57–59, 300–301). Thus, by drawing a circle on the screen and adjusting the instrument in such a way that the edge of the image should coincide everywhere with the circumference of the drawn circle, one could ascertain that the screen is indeed perpendicular to the cone of radiation. It is reasonable to expect

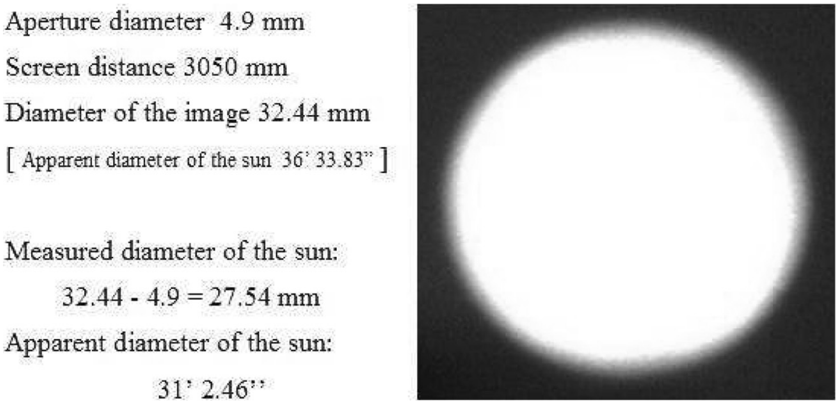

Figure 21. Photo of the image the Sun cast by a pinhole, 4.9 mm in diameter, placed 3050 mm from the screen, and the set of calculation by which the apparent diameter is determined.

that all the measured diameters of the Sun's images should be of the same size, as they are cast by the same source, namely, the Sun. But this consideration is idealized; in reality the results differ from each other, albeit by a small degree due to variations in marking the edges of the image.

Kepler sought to calculate the value of the apparent diameter of the Sun resulting from as many measurements as possible. In the case of the Moon, in addition to the measure taken on the screen, Kepler estimated its apparent diameter by a comparison with disks of various sizes which match the interior circumference of the Moon's image. Kepler's calculations of the apparent diameters of the Sun and the Moon yielded, however, slightly different results from each other. He therefore considered the mean of his calculations as the true apparent diameter of the Sun and Moon. Kepler's example of how the distance of the centers of the images at the beginning and end of eclipse is calculated suggests that he applied a procedure based on arithmetic mean to obtain the apparent diameter of the Sun and Moon (Donahue 2000, pp. 366–368, 426–428; *KGW* 2: pp. 304–306, 361–362).

What is the quality of Kepler's results? In figure 21 we can see the image of the Sun cast by a pinhole 4.9 mm in diameter placed 3050 mm from the screen. The measured diameter of the image is 32.44 mm which yields a solar apparent diameter of 36′ 33.83″. By Kepler's method, subtracting the diameter of the pinhole from the measured diameter of the

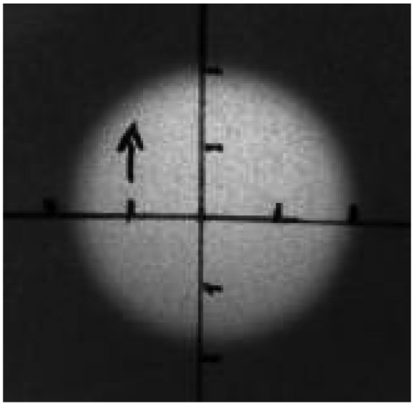

Figure 22. Photo of the cast image of the Sun projected through a pinhole of 3 mm in diameter

image gives 27.54 mm. The apparent diameter of the Sun, in this case, would be 31′ 2.46″.[16]

We evaluated the performance of a pinhole camera, with a pinhole of 3 mm in diameter placed 1931 mm from the screen.[17] Sequential measurements of the Sun's image (figure 22) were taken with a compass directly on the camera's screen and on printed photos as well. The mean apparent diameter of the Sun, measured and calculated similarly to what Kepler did, was 30′ 17.7″. We found that the difference between the extremes of twenty measurements of the cast image varies less than 2%.

Kepler's technique yielded a solar apparent diameter of 30′ at apogee and 31′ at perigee (Donahue 2000, p. 354; *KGW* 2: p. 294). As shown earlier, the diminution of the Sun's image projected through a 5.8 mm pinhole is about 4%. Consider the apparent diameter of the Sun at apogee, 31′ 30″, and at perigee, 32′ 30″, the results of Kepler's measurements should have been 30′ 14″ and 31′ 12″, respectively. The differences between Kepler's report of the apparent diameter of the Sun and our calculations is less than 1%. By comparison, according to Kepler, the measure-

16. The photo of the projected image of the Sun was taken on December 24, 2005. At the time, the true apparent diameter of the Sun was 32′ 31″.

17. The pinhole camera was mounted on a telescope at the Harry Kay Observatory (32° 26′ N; 34° 13′ E) on September 19, 2007. At the time, the true apparent diameter of the Sun was 31′ 50.52″. The measurements and photos were made when the Sun reached the highest point in the sky; that is, nearly 60 degrees above the horizon.

ments of the Sun's apparent diameter made by Tycho, were 29′ 36″ at apogee and 31′ 45″ at perigee (Donahue 2000, p. 354; *KGW* 2: p. 293). We conclude that for an experienced observer, the ecliptic instrument as described and operated by Kepler can produce consistent, reliable, and accurate results.[18]

Kepler was well aware of the discrepancies among the reports of his predecessors of the apparent diameters of the Sun and Moon. According to Kepler, these discrepancies were caused partly by different observer's eyes, partly by different light conditions, and partly by the difference in optical sights, which made the traditional procedures of observation using instruments untrustworthy (Donahue 2000, pp. 231–232; *KGW* 2: pp. 193–194).[19] To be sure, Kepler was aware of the limitations of his instrument. However, the technique of aiming the ecliptic instrument accurately and measuring the diameter of the image on the screen, as well as the method of taking readings on three different linear scales (on the azimutal quadrant, on the crosspiece, and on the rule), made Kepler's ecliptic instrument the most accurate astronomical device of its kind in the early years of the 17th century (Donahue 2000, pp. 358–359, 381–382, 423–429; *KGW* 2: pp. 297–298, 319, 356–362).

18. Sigismondi and Fraschetti (2001) presented an analysis of five measurements of the solar diameter made with a pinhole camera by Tycho (1591) and Kepler (1601–1602). The authors reproduced the measurements with a pinhole diameter of 6 mm. They located a plane mirror before the pinhole in order to orient the light horizontally, and to obtain a focal length, $f_{\text{(Focal length)}} \geq 20$ meter, which is well within the domain of far-field diffraction. The authors then surmised that, due to the diurnal motion of the Sun, Kepler's method could achieve no better accuracy than about 15 arc-seconds. They applied modern error analysis to Tycho's and Kepler's data, and concluded that the seasonal variation of the apparent solar diameter with a mean solar diameter of 1924″ ± 35″ at 95% confidence level is consistent with the actual mean solar diameter; that is, the pinhole camera is trustworthy. The authors claimed that Kepler's report of the smaller apparent diameters of the Sun, 30′ at apogee and 31′ at perigee, "is consistent with all the measurements made with pinhole that . . . [Kepler] examined, and also with the actual values of 31.5′ and 32.5′ once the systematic 'geometrical correction' is taken into account" (2001, 385). Principally, Tycho's and Kepler's pinhole cameras worked within the domain of near-field diffraction which affects the results of the measurements differently than the way it was considered by Sigismondi and Fraschetti. Based on far-field considerations, the authors missed the fact that a cast image, in near-field conditions, is reduced by about 4%. Thus, the small apparent diameters of the Sun, 30′ at apogee and 31′ at perigee, as reported by Kepler, is due to the diminution of the projected image, rather than to "systematic geometrical correction" which the authors claimed that Kepler had introduced into his calculations.

19. On the operational limits and accuracy of astronomical instruments of the time, see Chapman 1983; Roche 1981; Wesley 1979; Wesley 1978; and Thoren 1973, pp. 29–30, 40–42.

6. Conclusion

We revisited Kepler's study of the pinhole camera. Astronomical interest motivated Kepler's optical researches; his awareness of the problem of observational error informed the method and scope of these studies (Hon 1987). The measurements of the diameters of the Sun and the Moon when the images of these heavenly bodies are cast on a screen involved large errors. Kepler, therefore, endeavored first to gain a better understanding of the optical causes of the phenomenon of casting image and then to design a procedure for measuring the apparent diameter of what he called the luminaries, the Sun and the Moon.

To have a better understanding of the argument which Kepler developed, we recalled how in modern terms an image is formed behind a pinhole. We simulated and analyzed optical phenomena related to the working of the pinhole camera. With the knowledge of the operational limits and optical performance of the instrument, we can now explain the diminution suffered by an image projected by a pinhole system working at the domain of near-field diffraction; we calculated that the diminution is about 4% of the true apparent diameter of the Sun.

Furthermore, we discussed Kepler's theoretical insights and their applications in turning the pinhole camera into an astronomical instrument, as presented in chapter 11 of Kepler's *Paralipomena.* We followed Kepler identifying the sources of observational errors, seeking a solution, developing an experimental setup to test his argument, and finally putting forward an optical theory. Kepler thus established a geometrical framework within which he could manipulate and analyze the formation of pinhole images and execute two crucial calculations: (1) the apparent diameter of the Sun, and (2) the apparent diameter of the Sun when eclipsed.

To avoid the observational errors that had plagued the antiquated measuring techniques for calculating the apparent diameter and angular position of the luminaries, Kepler designed a novel device: an ecliptic instrument. The rule in this instrument is equipped with a pinhole camera which substitutes the traditional aiming device of slits-sight. The projected image on the screen of the camera has now two functions: (1) the shape of the image is used as a sensitive indicator for accurately aligning the rule with the Sun—the closer the shape of the image is to a circle, the better is the alignment; and (2) the cast image can be measured for the apparent diameter of the luminaries. He then carried out a series of calculations—based on the theory that he had developed—to determine the position and apparent diameter of the luminary in question. This procedure improved significantly the accuracy of the results.

We made astronomical observations with a modern replication of the pinhole camera, following closely Kepler's instructions, took measurements and calculated the apparent diameter of the Sun according to the method Kepler had developed. We confirmed that Kepler's observations and calculations of the apparent diameter of the Sun, which were $30'$ at apogee and $31'$ at perigee due to the diminution of the pinhole image, are accurate within the operational limits of the pinhole camera he had used. We conclude, then, that for an experienced observer a pinhole camera, as described and operated by Kepler, could produce consistent, reliable, and accurate results of the apparent diameters of the luminaries.

This close study of Kepler's theoretical insights into an optical apparatus, namely, the pinhole camera, and its combination with the quadrant to form the ecliptic instrument, sheds light on the role optical instruments play in Kepler's scheme: they impose constraints on theory, but at the same time render astronomical knowledge secure. To get a comprehensive grasp of Kepler's astonishing achievements it is required to widen the approach to his writings and examine his practice not only as a mathematico-physical astronomer, but also as an instrument maker and a practicing observer.

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