

Criteria for Robust Stability In A Class Of Lateral Inhibition Networks Coupled Through Resistive Grids

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In the analog VLSI implementation of neural systems, it is sometimes convenient to build lateral inhibition networks by using a locally connected on-chip resistive grid to interconnect active elements. A serious problem of unwanted spontaneous oscillation often arises with these circuits and renders them unusable in practice. This paper reports on criteria that guarantee these and certain other systems will be stable, even though the values of designed elements in the resistive grid may be imprecise and the location and values of parasitic elements may be unknown. The method is based on a rigorous, somewhat novel mathematical analysis using Tellegen's theorem (Penfield et al. 1970) from electrical circuits and the idea of a Popov multiplier (Vidyasagar 1978; Desoer and Vidyasagar 1975) from control theory. The criteria are *local* in that no overall analysis of the interconnected system is required for their use, *empirical* in that they involve only measurable frequency response data on the individual cells, and *robust* in that they are insensitive to network topology and to unmodelled parasitic resistances and capacitances in the interconnect network. Certain results are robust in the additional sense that specified *nonlinear* elements in the grid do not affect the stability criteria. The results are designed to be applicable, with further development, to complex and incompletely modelled living neural systems.

1 Introduction

In the VLSI implementation of lateral inhibition and certain other types of networks, active cells are locally interconnected through an on-chip resistive grid. Linear resistors fabricated in, e.g., polysilicon, could yield a very compact realization, and nonlinear resistive grids, made from MOS transistors, have been found useful for image segmentation (Hutchinson et al. 1988). Networks of this type can be divided into two classes: feedback systems and feedforward-only systems. In the feedforward case

one set of amplifiers imposes signal voltages or currents on the grid and another set reads out the resulting response for subsequent processing, while the same amplifiers both “write to” the grid and “read from” it in a feedback arrangement. Feedforward networks of this type are inherently stable, but feedback networks need not be.

A practical example is one of Mahowald and Mead’s retina chips (Mead and Mahowald 1988; Mead 1988) that achieve edge enhancement by means of lateral inhibition through a resistive grid. Figure 1a shows a single cell in an earlier version of this chip, and figure 1b illustrates the network of interconnected cells. Experiment has shown that the individual cells in this system are open-circuit stable and remain stable when the output of amplifier #2 is connected to a voltage source through a resistor, but the interconnected system oscillates so badly that the earlier design is scarcely usable¹ (Mahowald and Mead 1988). Such oscillations can readily occur in most resistive grid circuits with active elements and feedback, even when each individual cell is quite stable. Analysis of the conditions of instability by conventional methods appears hopeless, since the number of simultaneously active feedback loops is enormous.

This paper reports a practical design approach that rigorously guarantees such a system will be stable if the active cells meet certain criteria. The work begins with the naïve observation that the system would be stable if we could design each individual cell so that, although internally active, it acts like a passive system as seen from the resistive grid. The design goal in that case would be that each cell’s output impedance should be a *positive-real* (Vidyasagar 1978; Desoer and Vidyasagar 1975; Anderson and Vongpanitlerd 1973) function. This is sometimes possible in practice; we will show that the original network in figure 1a would satisfy this condition in the absence of certain parasitic elements. Furthermore, it is a condition one can verify experimentally by frequency-response measurements.

It is obvious that a collection of cells that appear passive at their terminals will form a stable system when interconnected through a passive medium such as a resistive grid, and that the stability of such a system is *robust* to perturbations by passive parasitic elements in the network. The work reported here goes beyond that observation to provide (i) a demonstration that the passivity or positive-real condition is much stronger than we actually need and that weaker conditions, more easily achieved in practice, suffice to guarantee robust stability of the linear active network model, and (ii) an extension of the analysis to the *nonlinear* domain that furthermore rules out sustained *large-signal* oscillations under certain conditions. A key feature of the integrated circuit environment that makes these results applicable is the almost total absence of on-chip inductance. While the *cells* can appear inductive, as in figure 3c,

¹The later design reported in (Mead and Mahowald 1988) avoids stability problems altogether, at a small cost in performance, by redesigning the circuits to passively sense the grid voltage in a “feedforward” style as described above.

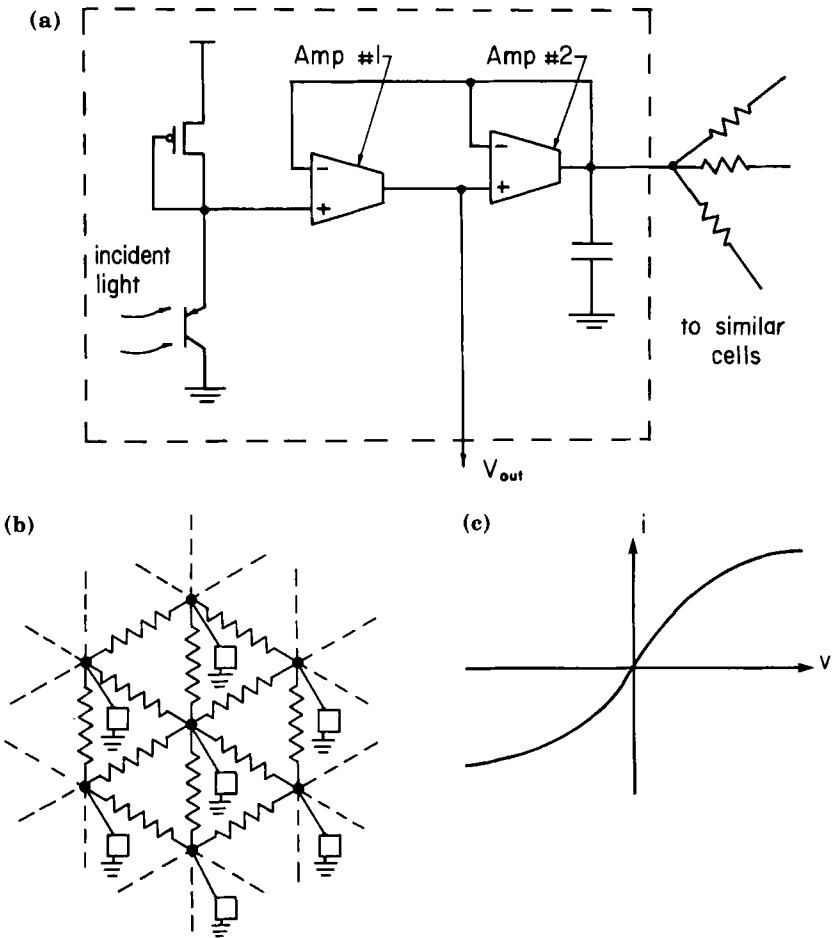


Figure 1: (a) This photoreceptor and signal processor circuit, using two MOS amplifiers, realizes spatial lateral inhibition and temporal sharpening by communicating with similar cells through a resistive grid. The resistors will often be nonlinear by design. (b) Interconnection of cells through a hexagonal resistive grid. Cells are drawn as 2-terminal elements with the power supply and signal output lines suppressed. The voltage on the capacitor in any given cell is affected both by the local light intensity incident on that cell and by the capacitor voltages on neighboring cells of identical design. The necessary ingredients for instability — active elements and signal feedback — are both present in this system. (c) Grid resistors with a nonlinear characteristic of the form $i = \tanh(v)$ can be useful in image segmentation (Hutchinson et al. 1988).

the absence of inductance in our *grid models* makes these theorems possible.

Note that these results do not apply directly to networks created by interconnecting neuron-like elements, as conventionally described in the literature on artificial neural systems. The “neurons” in, e.g., a Hopfield network (Hopfield 1984) are *unilateral 2-port elements* in which the input and output are both voltage signals. The input voltage uniquely and instantaneously determines the output voltage of such a neuron model, but the output can only affect the input via the resistive grid. In contrast, the cells in our system are *1-port electrical elements* (temporarily ignoring the optical input channel) in which the port voltage and port current are the two relevant signals, and each signal affects the other through the cell’s internal dynamics (modeled as a Thévenin equivalent impedance) as well as through the grid’s response.

It is apparent that uncontrolled spontaneous oscillation is a potential problem in living neural systems, which typically also consist of active elements arranged in feedback loops. Biological systems have surely solved *the same problem* we attack in this paper. It is reasonable to believe that stability has strongly constrained the set of network configurations nature has produced. Whatever Nature’s solutions may be, we suspect they have at least three features in common with the ones proposed here: (1) *robustness* in the face of wide component variation and the presence of parasitic network elements, (2) reliance on *empirical* data rather than anything we would recognize as a theory or analytic method, (3) stability strategies based on predominantly *local* information available to each network element.

Several reports on this work have appeared and will appear in (Wyatt and Standley 1988; Standley 1989; Standley and Wyatt 1989; 1988a; 1988b) during its development; a longer tutorial exposition will be given in the second printing of (Mead 1988).

2 The Linear Theory

2.1 Terminology. The *output impedance* of a linear system is a measure of the voltage response due to a change in output current while the input (light intensity in this case) is held constant. This standard electrical engineering concept will play a key role here. Figure 2a illustrates one experimental method for measuring the output impedance, and figure 2b is a standard graphical representation of an impedance, known as a *Nyquist diagram*. Similar plots have been used in experimental physiology (Cole 1932).

In the context of this work, an impedance is said to be *positive-real* (Vidyasagar 1978, Desoer and Vidyasagar 1975, Anderson and Vongpanitlerd 1973) if it is stable (i.e., has no poles or zeroes in the right-half plane) and its Nyquist diagram lies entirely in the right-half plane (i.e., in the

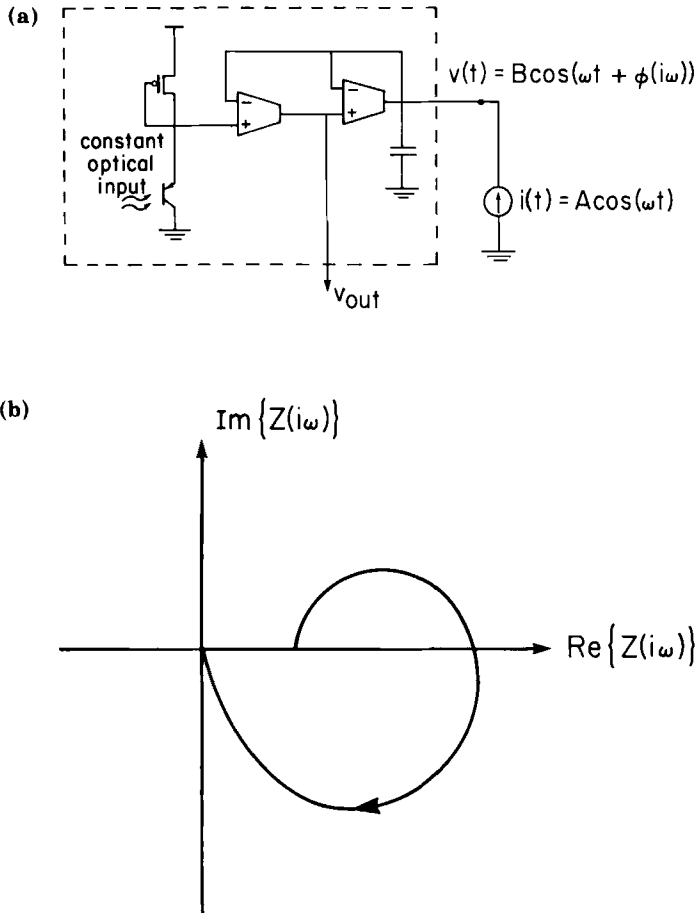


Figure 2: (a) Simplified experimental measurement of the output impedance of a cell. A sinusoidal current $i = A\cos(\omega t)$ is injected into the output and the voltage response $v = B\cos(\omega t + \phi)$ is measured. The impedance, which has magnitude B/A and phase ϕ , is typically treated as a complex number $Z(i\omega)$ that depends on the frequency ω . (b) Example of the *Nyquist diagram* of an impedance. This is a plot in the complex plane of the value of the impedance, measured or calculated at purely sinusoidal frequencies, ranging from zero upward toward infinity. It is not essential to think of Nyquist diagrams as representing complex numbers: they are simply polar plots in which radius represents impedance magnitude and angle to the horizontal axis represents phase. The diagram shown here is the Nyquist plot of the positive-real impedance in equation (2.1).

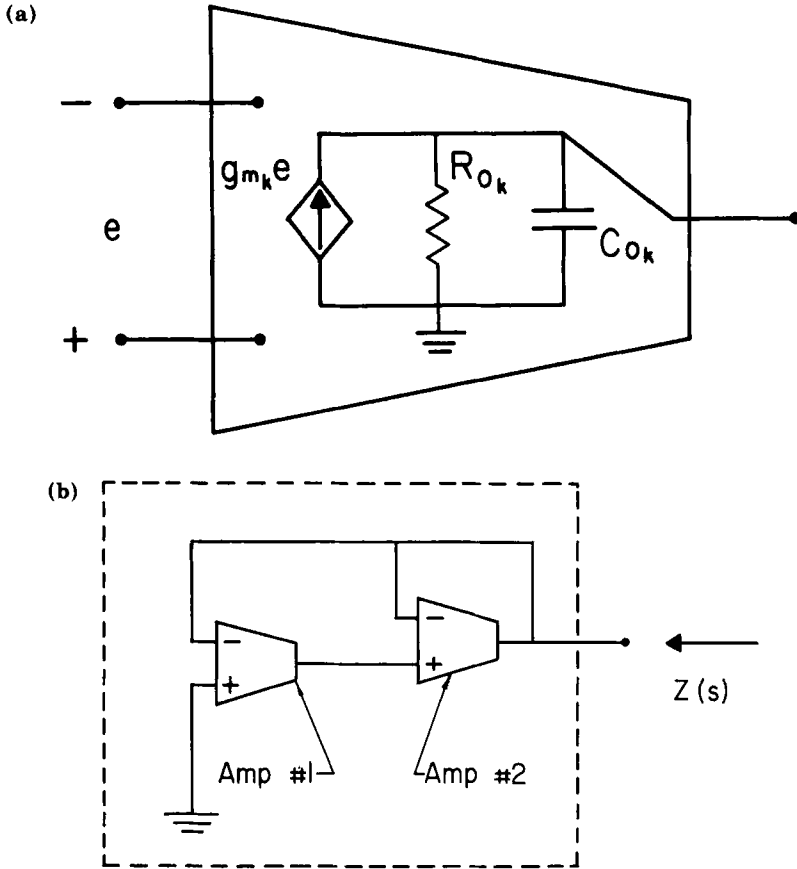


Figure 3: (a) Elementary model for an MOS amplifier. These amplifiers have a relatively high output resistance, which is determined by a bias setting (not shown). (b) Linearity allows this simplification of the network topology for the circuit in figure 1a without loss of information relevant to stability. The capacitor in figure 1a has been absorbed into the output capacitance of amp #2. (c) Passive network realization of the output impedance given in equation (2.1) for the network in (b).

language of complex numbers, $Re\{Z(i\omega)\} \geq 0$ for all purely sinusoidal frequencies ω). Figure 2a is an example, while the system represented in figure 4 is stable but not positive-real.

A deep link between positive-real functions, physical networks and

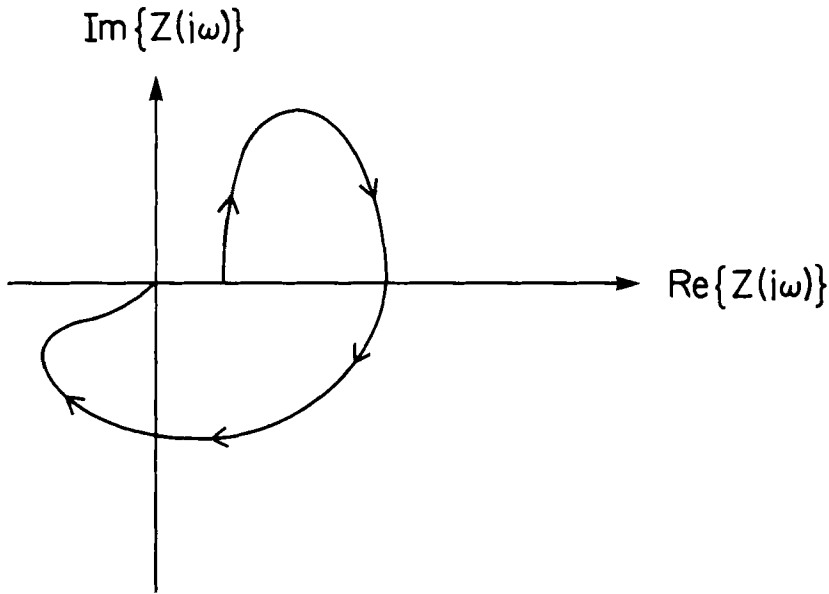


Figure 4: Nyquist diagram of an impedance that satisfies the *Popov criterion*, defined as follows. A linear impedance $Z(s)$ satisfies the Popov criterion if $(1 + \tau s)Z(s)$ is positive-real for some $\tau > 0$. The “Popov multiplier” $(1 + \tau s)$ modifies the Nyquist diagram by stretching and rotating it counterclockwise for $\omega > 0$. The impedance plotted here is active and thus is not positive-real, but the rotation due to the $(1 + \tau s)$ term can make it positive-real for an appropriate value of τ . The Popov criterion is a condition on the linear elements that is weaker than passivity: active elements satisfying this criterion are shown to pose no danger of instability even when nonlinear resistors and capacitors are present in the grid.

passivity is established by the classical result in linear circuit theory which states that $H(s)$ is positive-real if and only if it is possible to synthesize a 2-terminal network of positive linear resistors, capacitors, inductors and ideal transformers that has $H(s)$ as its driving-point impedance (Anderson and Vongpanitlerd 1973).

This work was originally motivated by the following linear analysis of a model for the circuit in figure 1a. For an initial approximation to the output impedance of the cell we use the elementary model shown in figure 3a for the amplifiers and simplify the circuit topology within a single cell as shown in figure 3b.

Straightforward calculations show that the output impedance is given by

$$Z^{-1}(s) = [g_{m_2} + R_{o_2}^{-1} + sC_{o_2}] + \frac{g_{m_1}g_{m_2}R_{o_1}}{(1 + sR_{o_1}C_{o_1})}. \quad (2.1)$$

This is a positive-real impedance that could be realized by a passive network of the form shown in figure 3c, where

$$R_1 = (g_{m_2} + R_{o_2}^{-1})^{-1}, R_2 = (g_{m_1}g_{m_2}R_{o_1})^{-1}, \text{ and } L = C_{o_1}/g_{m_1}g_{m_2}. \quad (2.2)$$

Of course this model is oversimplified, since the circuit *does* oscillate. Transistor parasitics and layout parasitics cause the output impedance of the individual active cells to deviate from the form given in equations (2.1) and (2.2), and any very accurate model will necessarily be quite high order. The following theorem shows how far one can relax the positive-real condition and still guarantee that the entire network is robustly stable. It obviously applies to a much wider range of linear networks than has been discussed here.

A linear network is said to be *stable* if for any initial condition the transient response converges asymptotically to a constant.

Theorem 1. Consider the class of linear networks of arbitrary topology, consisting of any number of positive 2-terminal resistors and capacitors and of N lumped linear impedances $Z_n(s)$, $n = 1, 2, \dots, N$, that are open- and short-circuit stable in isolation, i.e., that have no poles or zeroes in the closed right-half plane. Every such network is stable if at each frequency $\omega \geq 0$ there exists a phase angle $\theta(\omega)$ such that $0 \geq \theta(\omega) \geq -90^\circ$ and $|\angle Z_n(i\omega) - \theta(i\omega)| < 90^\circ$, $n = 1, 2, \dots, N$.

An equivalent statement of this last condition is that the Nyquist plot of each cell's output impedance for $\omega \geq 0$ never intersects the 2nd quadrant of the complex plane (figure 4 is an example), and that no two cells' output impedance phase angles can ever differ by as much as 180° . If all the active cells are designed identically and fabricated on the same chip, their phase angles should track fairly closely in practice, and thus this second condition is a natural one.

The theorem is intuitively reasonable and serves as a practical design goal. The assumptions guarantee that the cells cannot resonate with one another at any purely sinusoidal frequency $s = j\omega$ since their phase angles can never differ by as much as 180° , and they can never resonate with the resistors and capacitors since they can never appear simultaneously active and inductive at any sinusoidal frequency. A more advanced argument (Standley and Wyatt 1989) shows that exponentially growing instabilities are also ruled out.

3 Stability Result for Networks with Nonlinear Resistors and Capacitors

The previous results for linear networks can afford some limited insight into the behavior of nonlinear networks. If a linearized model is stable, then the equilibrium point of the original nonlinear network must be *locally stable*. But the result in this section, in contrast, applies to the full nonlinear circuit model and allows one to conclude that in certain circumstances the network cannot oscillate or otherwise fail to converge *even if the initial state is arbitrarily far from the equilibrium point*. Figure 4 introduces the *Popov criterion*, which is the basis of the following theorem. This is the first nonlinear result of its type that requires no assumptions on the network topology.

Theorem 2. Consider any network consisting of nonlinear resistors and capacitors and linear active cells with output impedances $Z_n(s)$, $n = 1, 2, \dots, N$. Suppose

- (a) the nonlinear resistor and capacitor characteristics, $i_j = g_j(v_j)$ and $q_k = h_k(v_k)$, respectively, are monotone increasing continuously differentiable functions, and
- (b) the impedances $Z_n(s)$ all satisfy the Popov criterion for some common value of $\tau > 0$.

Then the network is stable in the sense that, for any initial condition at $t = 0$,

$$\int_0^{\infty} \left[\sum_{\substack{\text{all resistors} \\ \text{and capacitors}}} i_k^2(t) \right] dt < \infty \quad (3.1)$$

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