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Composing by Laypeople: A Broader Perspective Provided by Arithmetic Operation Grammar

Abstract: Many existing approaches to teaching laypeople to compose are based, to a certain extent, on simply hiding the theoretical background. This is done, for example, by offering ready-made musical events that can be combined in any way to organize them in time. A different approach is taken in the work presented here. As an alternative to classical music theory, with all its complications, the generative composition method Arithmetic Operation Grammar (AOG) is used, which is much easier to learn. This approach has been taken in the conviction that the leaner theory on which it is based, in combination with the compact symbolic representation of entire compositions, can make a significant contribution to bringing forward the "everyday creativity" in the field of ubiquitous music. Furthermore, in the field of sonification, AOG offers the possibility of sonifying data that do not inherently include time as an ordering parameter. To prove practical use of this approach, AOG is combined with a user interface that is more suitable for adults as a target group and another one that is more appropriate for primary school children.

> There will always be great masterworks and great performances by individuals of exceptional knowledge and skill. . . . But anyone who thinks that such works should or will indefinitely maintain an exclusive dominance over all other musical forms and processes has not been paying much attention to musical history or ethnography, to what is going on musically in this world right now, or to what else—besides masterworks and virtuosi—people really do love about music (Spiegel, 1998).

The magic of performing a classical concert lies not least in the illusion of lightness and spontaneity with which a virtuoso on stage interacts with the orchestra. This is all, however, painstakingly worked out in years of musical study and even longer intensive practice of the musical instrument. Nowadays, impressive successes have been achieved in giving amateurs and children the opportunity to create real-time compositions themselves, and thus to have the exhilarating experience of becoming the creator of a perfect musical moment, so to speak. Current approaches typically focus on providing special user interfaces (UIs) that allow users to organize preproduced and preselected sound elements in time (Jakobsen et al. 2016; Stolfi et al. 2018; Figueiró, Soares, and Rohde 2019). This preselection ensures that the provided sound events can be combined arbitrarily. Alternately, some approaches postprocess the user interaction; nowadays more and more through the use of artificial intelligence (Biles 2007; Garcia-Valdez et al. 2013). The necessity to provide a theoretical background is thereby avoided. But the disadvantage of this procedure is also obvious: Once the given tonal material has been used extensively, it does not open up any further perspectives. Beyond an initial experience of what composing is, one remains, so to speak, caught in a hermetic world of sound with only limited possibilities. To move further beyond this point, the only option would be to deal with the traditional theoretical materials as supported, for example, by Almeida, Cabral, and Almeida (2019).

In the approach described here, a different path is taken. A method of composing will be introduced that also makes it possible to create compositions—even quite complex ones—that satisfy laws similar to those of the classical theory of harmony and counterpoint, but without having to refer directly to those laws or to any other music theory at all. The method presented here, called Arithmetic Operation Grammar (AOG), opens up an approach to composing for laypeople that does not require years of study, but does not need to hide the complexity of the mechanisms behind it, since these mechanisms are much simpler than those of classical music theory. The price for this convenience, however, is that it is not easy to emulate any existing musical style using AOG. The results achieved with AOG

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have certain qualities that have to be accepted as they stand.

Although AOG was presented for the first time in 2019 at the International Symposium on Computer Music Multidisciplinary Research (Kramann 2019), the focus will now be on the extent to which the use of AOG in combination with suitable UIs can simplify access to composing for laypeople. But what is meant by the claim that it is easier for amateurs to compose with AOG than on the basis of classical music theory? In slightly simplified terms, one can say that classical theory consists of a collection of rules and prohibitions. Following Noam Chomsky (1956, 1959), AOG represents a generative grammar of Type 3 . This means that the results are well-formed, which in turn means that in this case, apart from the generative rules of how music is generated from a symbolic representation, no further rules are needed to analyze and correct the result after its generation. Thus, AOG also takes a special position among generative grammars in relation to music: Typical representatives, such as Lindenmayer systems or cellular automata, produce patterns using symbolic representations before decisions have been made about how those patterns might be used musically. This means that these are extramusical procedures (Supper 2001), whereas AOG directly provides a musical structure that does not need further manipulation, beyond musical interpretation (instrumentation, playing techniques, dynamics, etc., which are still necessary). Thus AOG goes beyond other categories of generative grammar.

In an attempt to categorize AOG, it would also be conceivable, of course, to compare this technique with other approaches that use mathematical methods in some way to perform musical composition. These other approaches usually originate from studies of music theory in which mathematical models are created to put some aspects of composition into a more general context. Chord progressions are one such aspect. These approaches, typically, create topologies of all chord progressions that comply with a certain musical style. Choosing and traversing paths through these topologies can be seen as a rudimentary form of composing that involves this kind of modeling (see, e.g., Hu and Gerhard 2019).

The difference to this type of approach is that the mathematics in AOG do not represent a model of an existing type of music. Instead, special properties of a given mathematical object, namely, prime factorization of natural numbers, are exploited to create music. From the perspective of AOG the sequence of natural numbers contains an infinite number of intertwined melodies that are already in rhythmic and harmonic relationships with each other. These melodies are extracted by mathematical operations and made audible by a so-called selective division (explained in detail in the section "Arithmetic Operation Grammar"). By doing so, not only is a partial aspect of composing implemented, but also polyphonic compositions are generated in which melody, rhythm, and harmony are inherent.

If one would like to relate the compositional results achieved with AOG to any existing compositional styles, one is most likely to find correspondences in those 21st-century compositional directions that still essentially organize pitches in time but allow a broader spectrum of possibilities than is provided for in classical theory, such as the "tintinnabuli" harmony of Arvo Pärt, or in certain forms of free jazz. Rhythmic correspondences can be found above all in the repetitive elements of minimal music. But to get a first impression of the compositions that can be generated on the basis of AOG, refer to Section 1 of the thematically arranged selection of examples at http://kramann.info/98_AOG.

Arithmetic Operation Grammar

As mentioned above, AOG uses as a base element the natural numbers, considered as a time series. More precisely, this basic element consists of the nonnegative integers (including zero), in mathematical literature represented symbolically as $\mathbb{N}$ or $\mathbb{N}_0$, or any continuous finite subset. To show something like an inner musical organization of this sequence of numbers, a method will be used in the following to quantify the degree of dissonance of two integers. This method goes back to Leonhard Euler, who called it *gradus suivitatis*, written as $\boldsymbol{g}$ (Busch 1970).

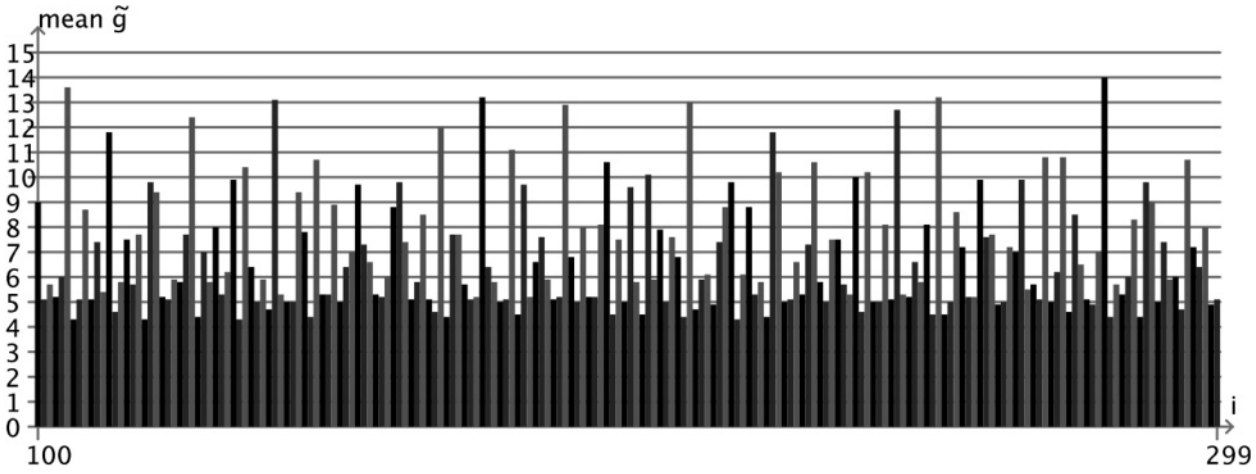


Figure 1. Mean value for $\tilde{\mathbf{g}}$ *between a number and its ten nearest neighbors (five smaller and five larger) in the range of natural numbers* i *from 100 to 299. Larger values tend to occur as individual peaks and not in clusters.*

To a certain degree, what is determined by this measure also corresponds to our auditory perception when the respective numbers are taken as oscillation periods or frequencies. The limitations of this method in terms of its transferability to human acoustic perception are deliberately accepted in this work in favor of the simplicity of the method and the fact that it works quite well over considerable ranges. It should be mentioned, however, that the two prime numbers 1,999 and 2,999, for example, provide a quite high value for $\boldsymbol{g}$, but we would hear them as a perfect fifth (ratio 2:3). For further information see, for example, papers by Schneider and Frieler (2008) and by Kramann (2015). In the following I will use the terms *gradus suivitatis* and degree of dissonance, understood in the musical sense, synonymously—aware that, from a musical point of view, I am not taking a subtly differentiated approach. But I am also not doing anything really out of the ordinary.

Calculating *gradus suivitatis*

Now, if the *gradus suivitatis* is to be determined for two integers a and b, one first factors out the greatest common divisor of a and b, then for the remaining prime factors p_i, one considers their powers k_i (the number of times the prime number is multiplied in the remaining factorization). According to Euler, the *gradus suivitatis* is then:

$$\boldsymbol{g} = 1 + \sum_{i=1}^{n} k_i \cdot (p_i - 1).$$

For example, for $a = 16 = 4 \cdot 2^2$ and $b = 20 = 4 \cdot 5^1$ (a major third), the result is

$$\boldsymbol{g}(16, 20) = 1 + 2 \cdot (2 - 1) + 1 \cdot (5 - 1) = 7.$$

For $a = 36 = 6 \cdot 2^1 \cdot 3^1$ and $b = 30 = 6 \cdot 5^1$ (a minor third), the result is

$$\boldsymbol{g}(36, 30) = 1 + 1 \cdot (2 - 1) + 1 \cdot (3 - 1) + 1 \cdot (5 - 1) = 8.$$

If one now considers the *gradus suivitatis* between a selected natural number and its closest neighbors, it is clear that as the selected number increases, larger and larger values of $\boldsymbol{g}$ will also tend to occur, since the larger the selected number and its neighbors are, the larger are their prime factors, on average.

If we use a modified *gradus* $\tilde{\mathbf{g}}$ limited to the first four primes (2, 3, 5, and 7), however, and if we limit the exponents used with these prime factors, a different picture emerges. The larger the value of the modified averaged $\tilde{\boldsymbol{g}}_{\boldsymbol{mean}}$, the less often it occurs for any number. Additionally, it is typically the case that large $\tilde{\boldsymbol{g}}_{\boldsymbol{mean}}$ values are surrounded by small ones. They tend to appear as single peaks and not in clusters (see Figure 1).

The modified function $\tilde{\mathbf{g}}$ can be defined for two natural numbers a and b as follows: If u is the greatest common denominator of both numbers, it follows that $a = u \cdot x$ and $b = u \cdot y$. The prime factors that are not common to both numbers are then $c = x \cdot y = 2^p \cdot 3^q \cdot 5^r \cdot 7^s \cdot \text{REST}$, where REST refers to the product of all the prime factors greater than 7. To calculate $\tilde{\mathbf{g}}$, upper limits are defined for p, q, r and s. These capped parameters are denoted $\bar{p}$, $\bar{q}$, $\bar{r}$, and $\bar{s}$. As upper limits for the exponents we use $\bar{p}_{max} = 3$, $\bar{q}_{max} = 2$, $\bar{r}_{max} = 1$ and $\bar{s}_{max} = 1$. For example, if the value of q is 4, then only 2 may be transferred to $\bar{q}$. For the modified *gradus*, we obtain

$$\tilde{\mathbf{g}}(a, b) = 1 + \bar{p} \cdot (2-1) + \bar{q} \cdot (3-1) + \bar{r} \cdot (5-1) + \bar{s} \cdot (7-1).$$

See Figure 1, in which, for every number in the range 100 to 299, these limited *gradus suivitatis* are calculated with the five left and five right neighboring values, and the mean of all ten values is shown as a bar.

Using the Modified *gradus suivitatis*

A preliminary motivation for this limited *gradus suivitatis* is that in music (insofar as it is an organization of sound events of defined pitch in time), rather small prime factors play a formative role, concerning both the frequency ratios of musical intervals and the rhythm. But if, with the help of the limited *gradus suivitatis*, what could be called this humanized perspective on the natural numbers, something remarkable occurs: The fact that numbers that are more dissonant with the numbers surrounding them tend to occur as individual peaks, and not in clusters, corresponds quite well with the common musical rules of traditional Western composition. For example, in classical choral composition, roughly speaking, any number of consonances may be strung together, but dissonances should always alternate with consonances. This means that they should occur less frequently and be well distributed throughout the movement. (Recall that AOG uses the natural numbers as a time series.)

To make this tendency even more visible, consider Figure 2. This time, the points entered in this semilogarithmic graph represent all natural numbers in the range between 100 and 5,100. The *x*-axis in this graph corresponds to the mean value for $\tilde{\mathbf{g}}$ between the number in question and its 180 closest neighbors (90 smaller and 90 larger). The *y*-axis is a logarithmic representation of how far from the current number the next number with the same or greater value of $\tilde{\mathbf{g}}_{mean}$ is. This selection was made as a relatively arbitrary example. It has been shown, however, that the basic structure visible here remains intact, even if changes are made to the range of numbers shown, or the number of neighbors considered. In the graph, one can see a band of points lying diagonally with a positive gradient. This can be interpreted to mean that as the value of $\tilde{\mathbf{g}}_{mean}$ increases, the distance to a next value with at least the same value of $\tilde{\mathbf{g}}_{mean}$ actually increases exponentially. This corresponds to the previous statement that the greater the corresponding $\tilde{\mathbf{g}}_{mean}$, the more isolated the values.

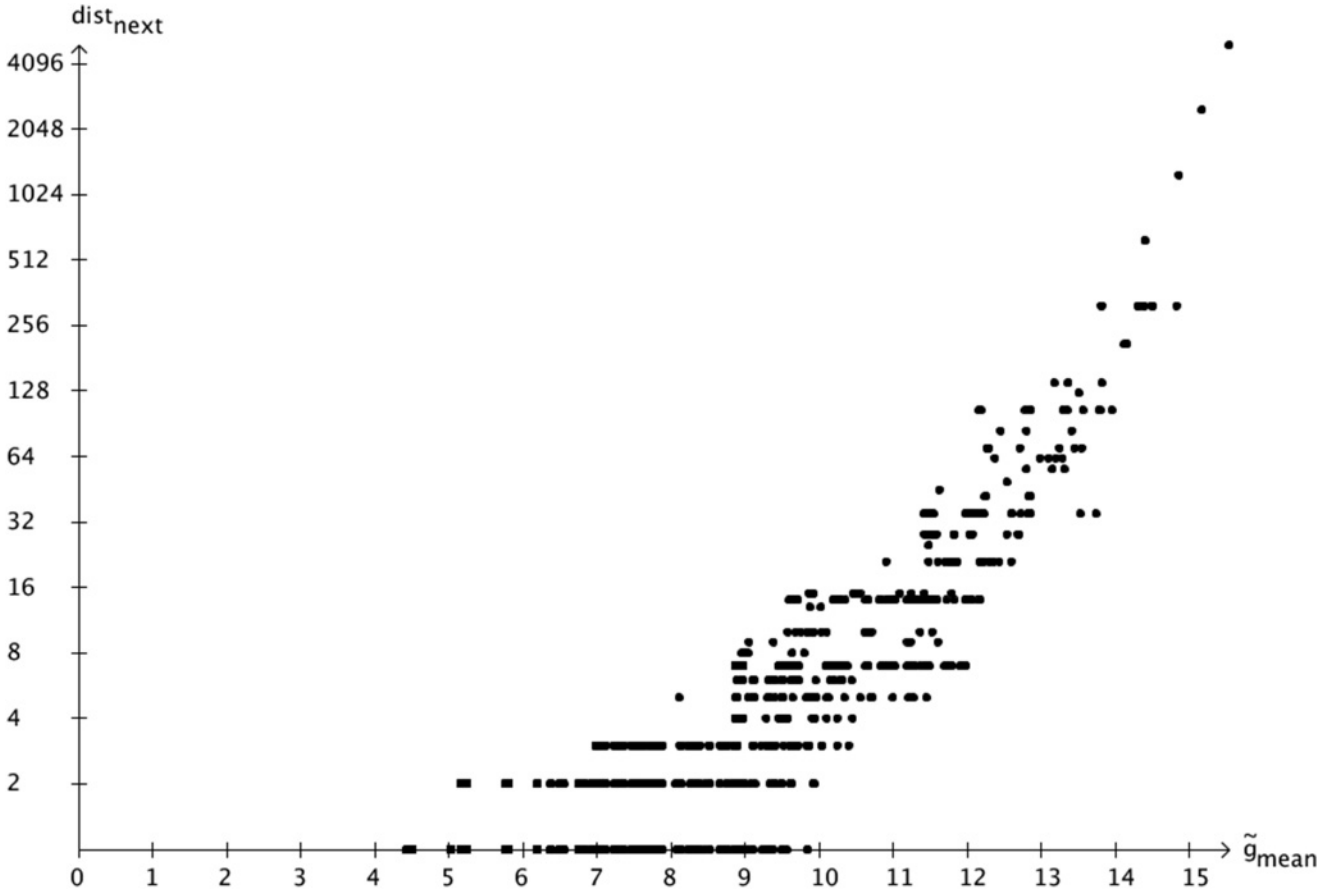
Or, seeing $\mathbb{N}$ as a time series and interpreted musically: Tones that lie in a dissonant relationship to other tones nearby in time occur sporadically and increasingly rarely, the greater this dissonance value is on average.

Composing with Natural Numbers

Here then, in the form of this inner structure of natural numbers from the perspective of $\tilde{\mathbf{g}}$, we have the basic element mentioned above, which already satisfies musical laws. Remarkably, this is an element that did not need to be invented but only discovered, and which is infinitely large and diverse. By exercising restraint in constructing or creatively designing a basic element, one has been revealed that is literally ubiquitous and so corresponds to the ideal of ubiquitous music on an unexpected level.

The next step is to show how this basic element can be represented audibly. The source code in Figure 3 simply goes through the natural numbers and filters the powers of the first four prime numbers out of each number. As in $\tilde{\mathbf{g}}$, the respective exponents that are still considered are also capped here. This is done with a kind of selective division. For this purpose, first the number B is calculated as the

Figure 2. Correlation between $\tilde{\mathbf{g}}_{\text{mean}}$ and the distance to the next number whose $\tilde{\mathbf{g}}_{\text{mean}}$ is at least as large; see text for further details.



product of the maximum powers of the individual prime numbers 2, 3, 5, and 7. In the example here, $B = 2^3 \cdot 3^2 \cdot 5^1 \cdot 7^1 = 2{,}520$. Each natural number currently under consideration divides B only with the prime factors 2, 3, 5, and 7, and this with a maximum exponent of 3 for the number 2, a maximum exponent of 2 for 3, and a maximum exponent of 1 for both 5 and 7. The symbol // will be used for this selective division.

A few concrete examples for clarificatation:

$$B//12 = 2 \cdot 3 \cdot 5 \cdot 7$$
$$B//35 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$$
$$B//13 = B$$
$$B//16 = 3 \cdot 3 \cdot 5 \cdot 7.$$

In the last example, $16 = 2^4$ exceeds the maximum power allowed for the prime 2, i.e., $2^3 = 8$, so only 8 is used in selective division; in other words, $B//16 = B//8$. The result of this selective division is rendered as a note of that frequency and is played if this frequency is inside the range of the musical instrument (acoustic or electronic) being used. The program was implemented in Processing/Java, see the example "BASIC_Sound_of_N" in the contributed library "ComposingForEveryone" at https://www.processing.org. In the interest of keeping the examples brief, no effort was made to achieve a particularly interesting sound. A more elaborate realization of the same principle can be seen in the YouTube video https://youtu.be/e81wd1b3FEE.

Other pieces of music are now created by changing this basic element through the application of

Figure 3. Simple sonification of the natural numbers.

Figure 4. A short phrase composed using AOG.

```
int BASENUMBER = 2*2*2*3*3*5*7;
/*t: finite section of the natural numbers as a time series*/
for(int t=0;t<1000;t++)
{
int f = BASENUMBER;
/*Selective division: Extract prime factors 2,3,5,7 from t*/
/*and factor them out of the BASENUMBER (% is modulo):*/
while(t>=2 && t%2==0 && f>=2 && f%2==0) {t=t/2;f=f/2;}
while(t>=3 && t%3==0 && f>=3 && f%3==0) {t=t/3;f=f/3;}
while(t>=5 && t%5==0 && f>=5 && f%5==0) {t=t/5;f=f/5;}
while(t>=7 && t%7==0 && f>=7 && f%7==0) {t=t/7;f=f/7;}
/*Interpret the result as a frequency and play it:*/
if(f>=55 && f<=1760) play(f);
delay(200); /*time delay of e.g., 200 milliseconds*/
}
```

Figure 3

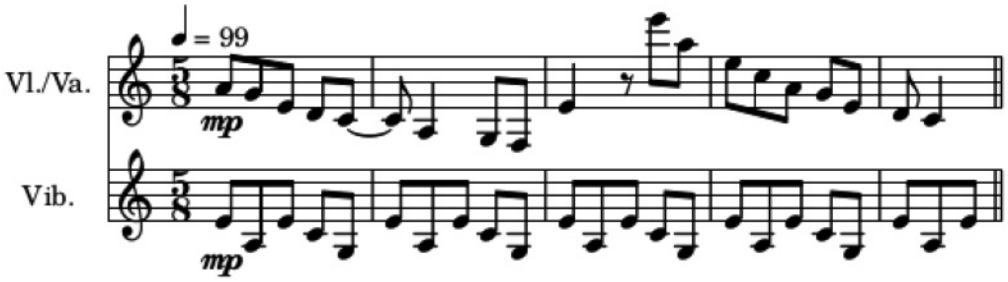


Figure 4

arithmetic operations, including modulo division, and also using selective division for the resulting sequence elements to obtain a temporal progression of pitches. In AOG, "composing" means defining formulas according to which the basic element $id(\mathbb{N})$ is modified. (The notation $id(\mathbb{N})$ means "identity of $\mathbb{N}$," a mathematical expression stating that $\mathbb{N}$ is mapped to itself, i.e., it forms a series. In an algorithmic terminology this corresponds to the succession function.) At least for the basic arithmetic operations, the resulting sequence also satisfies the Peano axioms and so retains all the properties of the natural numbers, including the musical ones (Russell 1920, pp. 1–10). In other words: the basic structure visualized in Figure 2 remains similar for these modifications of the basic approach.

But for the operation of division, this is only the case if one excludes all the division results that have a remainder. Several formulas can be used simultaneously to generate multiple voices. This will be illustrated using a simple, concrete example. The two formulas

$$f_1 = B//((t + 16) \bmod 17)$$
$$f_2 = B//(((t * 34) \bmod 10) + 8)$$

with $B = 2{,}520$ and $t = 9{,}000 \ldots 9{,}022$ provide the two-part musical phrase shown in Figure 4.

The generally occurring harmonic and contrapuntal interrelationships between two or more voices that are generated by AOG and run polyphonically have not yet been systematically investigated in my work. One could argue that in AOG the same sequence t, which is an extract of $id(\mathbb{N})$ for all voices, is the starting point for the following operations. In t all divisors occur with a fixed period that is equal to their value. For example, the divisor 3 has a period of 3, as in the sequence 0 . . 3 . . 6 . . 9 . . , etc. Considering this fact, the preservation of a harmonically meaningful relationship can now be shown, for at least some special cases, to make the existence of a general tendency for this at least plausible: When applying multiplications by numbers consisting only of prime factors greater than 7, the relevant part of the factorization-structure of the resulting sequence does not change with regard to the original sequence before multiplication. Multiplications with small prime factors, like 2 or 3, create a rather consonant relation of elements of both sequences, which are close in time.

For example,

$$
\begin{aligned}
t &= \{\ldots 6, 7, 8, 9, 10, 11, 12, \ldots\};\\
f_1 &= 2520//t\\
&= \{\ldots 420, 360, 315, 280, 252, 0, 210, \ldots\};\\
2t &= \{\ldots 12, 14, 16, 18, 20, 22, 24, \ldots\};\\
f_2 &= 2520//(2t)\\
&= \{\ldots 210, 180, 315, 140, 126, 1260, 105, \ldots\}
\end{aligned}
$$

where 2,520 is the base number B. The values for f_1 and f_2 are frequencies in Hertz. The multiplication here by two results, in most cases, in a transposition by an octave downwards. One exception is the interval of a unison for the frequency 315, because, as noted earlier, $B//16 = B//8$. Another exception is the next-to-last values in the two sequences, $f_1 = 2520//11$ and $f_2 = 2520//2$. In the case of f_1 the selective division is not possible and the resulting zero means that no note is played. In contrast, selective division becomes possible at this point in sequence f_2, after the multiplication by two, and yields 1,260 Hz.

Similarly, rather consonant relationships are created by the addition of numbers that are rich in the prime factors 2, 3, 5, and 7, because such shifts are widely in phase with the rhythms in which just these divisors appear. Following on from the previous example,

$$
\begin{aligned}
t + 6 &= \{\ldots 12, 13, 14, 15, 16, 17, 18, \ldots\};\\
f_3 &= 2520//(t + 6)\\
&= \{\ldots 210, 0, 180, 168, 315, 0, 140 \ldots\}.
\end{aligned}
$$

In f_3 the 168 is "new," but harmonizes well, since $252 = 168 \cdot (3/2)$, a perfect fifth.

Composing for Everyone

Figure 5 supplies the MIDI pitches from the formulas that correspond to the notes shown in the score of Figure 4. A musical phrase (melody, rhythm, and harmony) is created from these few lines of code. Of course, in program code intended for an application, the formulas would not be hardcoded. Instead, they could be constantly changed by the user with the aid of an editor, as for example in a corresponding Android app freely available on Google Play (https://play.google.com/store/apps/details?id=info.kramann.cfe, see Figure 6). For the sake of compactness, this editor does not use parentheses, nor does it display the selective division that is always performed. The fact that the operations are applied to $t = id(\mathbb{N})$ is not displayed. This example could be displayed in the app simply as "$+16 \equiv 17$" and "$\cdot 34 \equiv 10 + 8$" (in the editor the symbol $\equiv$ is used for modulo division).

Having now introduced AOG by means of an example, it will now be shown at which points specifications were made that not only apply to the specific example but can generally be varied to change the characteristic of the generated music. We will also look at the meaning of the operations from a musical point of view. First, it should be noted that, despite the effort made here to demonstrate the relationship between AOG and traditional compositional methods, the method itself is quite simple. The example shown here could easily be reproduced with paper and pencil, apart from the

Figure 5. Composing a small phrase on the basis of AOG. Transposition up by a half step results as no accidentals are necessary for the representation of the notes. This clarifies the diatonic character of the tone scale resulting from $B = 2{,}520$.

```
int B = 2520; /*base number*/
for(int t=9000;t<=9022;t++) /*Extract from the natural numbers*/
{
int x = (t+16)%17; /*Formula 1*/
int y = ((t*34)%10)+8; /*Formula 2*/
int f1 = B;
/*Selective division 1:*/
while(f1>=2 && f1%2==0 && x>=2 && x%2==0) {f1=f1/2;x=x/2;}
while(f1>=3 && f1%3==0 && x>=3 && x%3==0) {f1=f1/3;x=x/3;}
while(f1>=5 && f1%5==0 && x>=5 && x%5==0) {f1=f1/5;x=x/5;}
while(f1>=7 && f1%7==0 && x>=7 && x%7==0) {f1=f1/7;x=x/7;}
int f2 = B;
/*Selective division 2:*/
while(f2>=2 && f2%2==0 && y>=2 && y%2==0) {f2=f2/2;y=y/2;}
while(f2>=3 && f2%3==0 && y>=3 && y%3==0) {f2=f2/3;y=y/3;}
while(f2>=5 && f2%5==0 && y>=5 && y%5==0) {f2=f2/5;y=y/5;}
while(f2>=7 && f2%7==0 && y>=7 && y%7==0) {f2=f2/7;y=y/7;}
/*Mapping the resulting frequencies to midi pitches:*/
float factor = pow(2.0, 69.0/12.0)/440.0;
int midi1 = (int)round(12.0*log((float)f1*factor)/log(2.0));
int midi2 = (int)round(12.0*log((float)f2*factor)/log(2.0));
/*midi==0 means no sound or continue previous tone.*/
/*Otherwise transpose a half tone upwards to avoid accidentals*/
if(midi1<52 || midi1>89) midi1=0; else midi1=midi1+1;
if(midi2<52 || midi2>89) midi2=0; else midi2=midi2+1;
/*Simple output of the results on the terminal for checking:*/
println(t+" "+midi1+" "+midi2);
}
```

quantization due to the tempered scale. This fact is emphasized here, since the goal was to present a substitute for classical music theory that would be easier to learn. It should not be misunderstood, however, that the reason why most people will find it easy to learn and understand this method is that they have already learned arithmetic in their school years and have practiced the use of equation systems. But even if one would have to learn all this to use AOG, the gain would still be the highly compact representation of a composition using algebraic expressions and the possibility to change the character of a composition in its entirety just by changing a few symbols. The example given here is brief. A piece described by AOG is, however, potentially infinitely long and diverse, because the time sequence of natural numbers forming the input can be continued infinitely, supplying ever-new variants of sequences of successive powers of prime factors.

Composing with AOGdogma

To avoid the impression that all pieces created with AOG have the minimalist style of the example in Figure 4, at least one more piece should be presented as a contrast, a piece that is much more demanding in its musical conception, that uses more than a pentatonic scale, and in which the instruments are playing with many pitch classes. The latter is achieved by the fact that the base number changes during the course of the piece (see Figure 7 and the demonstration at https://youtu.be/BYzr9RpfFhc).

User-Specified Attributes of AOG

The character of compositions created using AOG is determined by specifying the following elements.

First, the user can specify the number of sources of $t = id(\mathbb{N})$ to be used.

Figure 6. The Android app CFE (Composing for Everyone) is a direct implementation of AOG and can be obtained on Google Play. An AOG formula can be entered for each of four addressable musical instruments using the on-screen keyboard. The resulting piano roll is displayed in the lower portion of the screen to provide direct feedback. In addition to the basic arithmetic operations and modulo, experimental operations are available that support direct manipulation of the powers of the prime factors 2, 3, 5, and 7. Each of the upper four lines in the editor controls one of four musical instruments. With the entries to be seen and in the symbolic notation introduced here, the frequencies passed through by these instruments are then calculated as follows: $f_1 = B//(t + 6)$, $f_2 = B//(t + 12)$, $f_3 = B//(t + 18)$, *and* $f_4 = B//(t + 24)$.

Second, the user can configure the operations to be applied to each t_i, and the order in which they are applied.

Third, the user can specify how to treat operations that are inapplicable or invalid in certain conditions. For example, a pending division by zero could either result in not playing a note at all on the current tick due to the formula in which the operation occurs, or the operation could simply be skipped. Instead of omitting decimal places after a division, the operation could be skipped when the division occurs, and so on.

Fourth, in the chain of operations applied to each integer in the series, the user can specify at which operations a sound conversion can take place through application of selective division. If the result after each operation is used to produce sounds, one obtains exactly as many voices as there are operations. This can be used directly with polyphonic instruments, such as the piano, to control their dynamics. If the same frequency appears as a result of several operations simultaneously, these can overlap to form a louder tone. Alternatively, one can proceed in such a way that, along a sequence of operations, only the last playable tone is heard. In this case, the number of voices is identical to the number of active formulas.

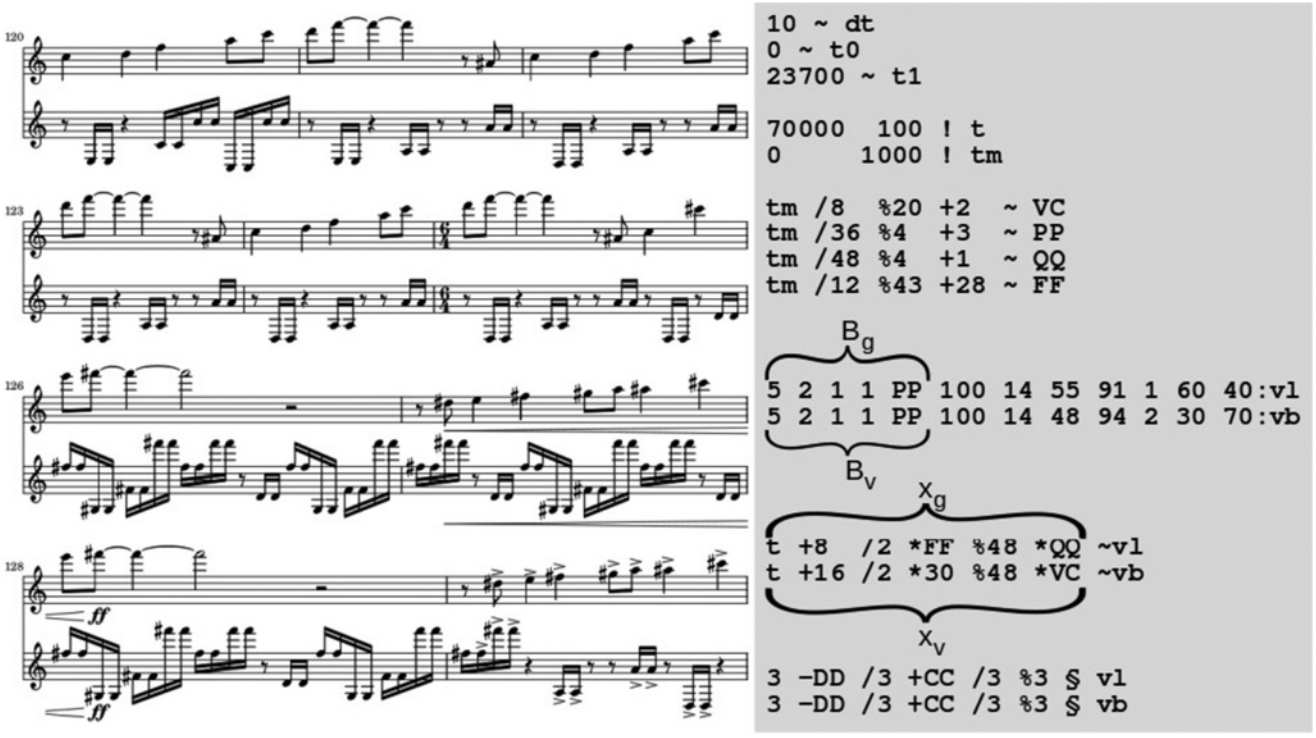
The fifth user-configurable parameter is the base number. This value is mainly responsible for which frequencies can be generated with the respective AOG mechanism. At this point it bears repeating that x, which is a result of applying arithmetic operations to the sequence of natural numbers, for example $x = t/7 + 3$, appears in the denominator in selective division. The base number is the numerator. To limit the type and count of the prime factors considered, it is possible to extract them directly from x and interpret this result as frequency instead of introducing selective division. This can then be expressed symbolically as $f = B ** x$. In fact, both operations are used in the somewhat more-sophisticated composition "Elegie" (see Section 1 at http://kramann.info/98_AOG). If $f = B//x$ leads to no playable result, but $f = B ** x$ does, the latter is used. An advantage of the former (selective division) compared to the latter (pure selection) is that even a very small x results in a playable frequency. The harmonic relations of two numbers do not change, regardless if they are taken directly or as reciprocal values.

A sixth parameter is whether to directly use the integer frequencies calculated, or to quantize these frequencies to equal temperament. In the latter case, the frequency range can be compressed or stretched by a certain factor before this quantization, and after that the entire composition can be transposed at will, as, for example, in the example above by a half step upwards (as in Figure 5). All three elements together, that is, the base number, a stretch factor, and a transposition, can be used to produce a certain desired musical scale. This scale can also be microtonal.

Additionally, all of the above elements can be changed over time—slowly, relative to the time sequence $t = id(\mathbb{N})$.

Finally, in the postprocessing layer there should be an algorithm for the musical interpretation of the incoming notes to be played. Concretely, a mechanism has been implemented that uses the *gravus suivitatis* between the current tone and the n immediately preceding and all simultaneously sounding tones to adjust its playing technique (staccato, legato, etc.) and dynamics. What is more, commercial physical modeling software was used for the sound generation. All the pieces of music to which reference was made at the beginning and some more

Figure 7. Excerpt from the score and its AOG representation of the composition AOGdogma#3, which was generated using the real-time composition framework AOGdogma (www.kramann.info/90_AOGdogma). It is a composition for violin and vibraphone lasting four minutes. The sequence of operations applied to the parameter t for the violin are represented by x_g, those for the vibraphone by x_v. Thus the frequencies that are passed through on both instruments are calculated as $f_g = B_g//x_g$ and $f_v = B_v//x_v$. The following applies here: $B_g = B_v = 2^5 \cdot 3^2 \cdot 5^1 \cdot 7^1 \cdot$ PP. The parameters VC, PP, QQ, and FF "drift," that is, they provide a slow metamorphosis of both the two AOG formulas and their base numbers. The parameter t is the extract of the natural numbers that are passed through starting with 70,000 in steps of one with an equidistant time interval of 100 milliseconds; tm is a counting process that proceeds more slowly and controls the drifting parameters. By adding a language element for the description of drifting parameters and the possibility to use it at any position of the AOG formulas, and by providing this together with an editor for real-time composition, AOGdogma makes it possible to compactly represent whole compositions and to create them interactively.



are representatives of this approach (See Sections 1, 4, 5, and 7 at http://kramann.info/98_AOG).

AOG and Traditional Musical Forms

For the effect of the individual operations, direct correspondences can, in some cases, be shown to certain traditional musical forms:

1. Addition. This brings a sound event forward in time. An addition of 3 maps the sequence {0, 1, 2, 3, 4, 5} to {3, 4, 5, 6, 7, 8}. If +3 is entered for one voice and +6 for the other, this results in a kind of two-part canon. After these operations, selective division followed by a rendering of the resulting number as a note of that frequency is, of course, always performed. In the formula notation introduced, one then has a melody in the simplest case, resulting from $f_i = B//(t_i + 3)$, and a second melody resulting from $\tilde{f}_i = B//(t_i + 6)$, whereby the latter is three ticks ahead of the former. The following points should be understood in the same sense.
2. Divisions. Compared to the unchanged voice, when for instance a division by two is performed, the new voice is twice as slow. This corresponds to the musical procedure of canon by augmentation.
3. Modulo or Remainder-after-Division. The result of this operation is always the integer

remainder of a division using the same operand. Modulo 6, for example, maps the sequence {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} to the sequence {0, 1, 2, 3, 4, 5, 0, 1, 2, 3}. The musical structures that arise are reminiscent of the repetitive phrases typically found in minimal music.

User Interfaces

As examples, two UIs are described below, both of which are based on AOG. The first is a formula editor, more suitable for adults. The second is a system in which compositions are created by decorating balls with colored tape, beads, pompoms, and other items. The latter interface is intended for use by primary school children.

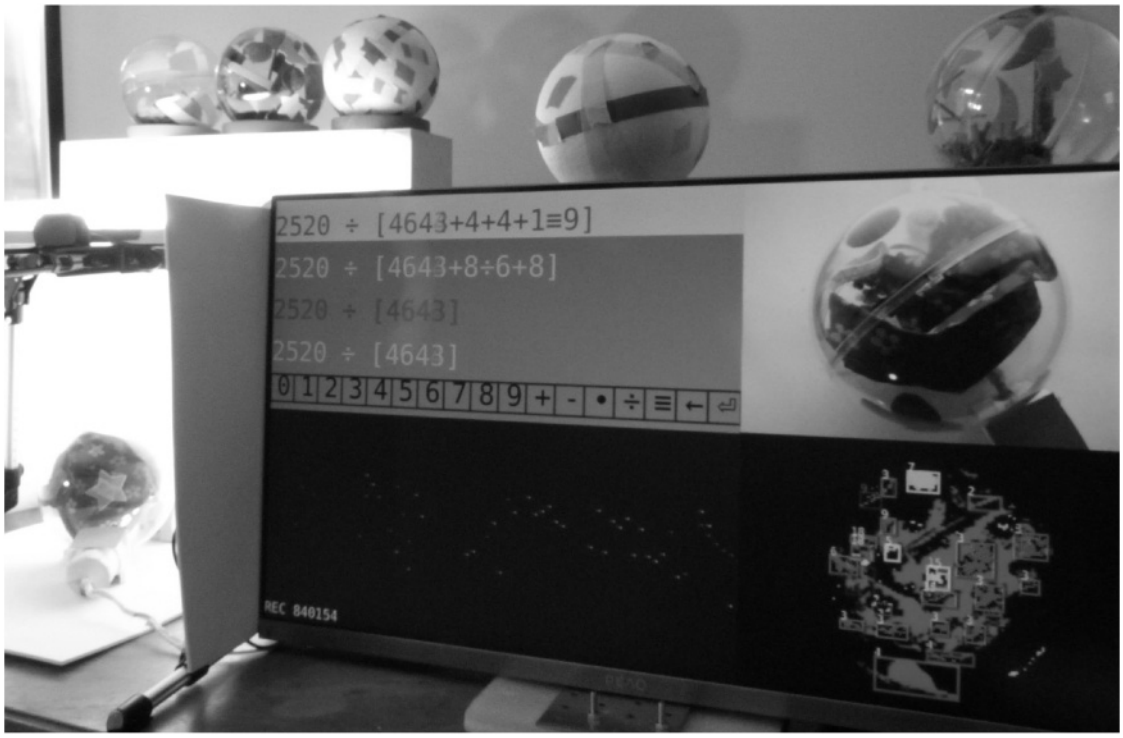
The formula editor was tried by attendees of a TEDx event (see Section 2 at http://kramann.info/98_AOG). In groups of four attendees at a time, people could enter AOG operations using four Android tablets, with each tablet assigned to a voice. The UI design is similar to that of the Android app seen in Figure 6. The formulas entered were converted into music according to the AOG method by a personal computer connected to all tablets over a wireless local area network. Likewise, all currently valid formulas were displayed to all those present with the aid of a projector. The four formulas were displayed in color-coded form and, based on a previous introduction, the participants should have been familiar with the link between a formula and the musical instrument assigned to it, whose individual sound was chosen to be distinct from each of the other three instruments. Through this feedback a certain exchange of information was established among the participants after some time, and a certain learning effect could be observed: The participants started to copy other participants' formulas if they liked the result and then experimented again with variations of this basic form. The functions of the editor and the basic effects of the individual operations on the musical result were explained to the audience in the previous lecture. To what extent the lecture was consciously referred to could not be determined within the 20 minutes in which the audience experiment took place.

The observed casual collaboration by "copy and variation" was possible with the system because the symbolic representation of the resulting composition is so compact that it could be grasped at a glance and quickly adopted.

The promotion of this type of cooperation by copy and variation through an easy-to-capture form of feedback is also a central feature of the second UI presented here, called "The Flippin' Pompoms" (TFP). The role of the formulas, which are visible to all in the formula editor, is played in TFP by colorfully decorated balls, which are kept visible to all, with each ball representing a composition. Some examples are shown in Figure 8. More examples, including a video of the system in action, can be found at Section 5 on http://kramann.info/98_AOG. Due to the COVID-19 pandemic, however, the practical experiences with TFP have so far been limited to small example experiments with children. In particular, it had been planned to use this system in a STEM project day at a primary school; this plan has had to be postponed indefinitely.

The balls can be designed according to compositional considerations. For example, there is a ball in which a pompom is pasted with colored areas. Whenever the pompom flips, a new variant of the musical structure appears. The resulting balls are placed on a motorized support that slowly rotates the balls around two axes, while a webcam continuously records the changing view and passes it on to the software as a pixel image. Because this textual description may give an overview of the functionality, despite not being possible to reproduce every detail exactly, the source code for TFP was provided as an example within the contributed Processing Library "ComposingForEveryone" (see Section 9 at http://kramann.info/98_AOG). In this software the image is filtered in such a way that there is only a black background and colored shapes in red, green, blue, and yellow (Figure 9a). In the next step, single-colored contiguous areas are segmented. Several such areas can be connected again and form a colorful contiguous area. Alternately, they may exist separately without connection to other areas. For areas that are composed of several colored areas,

Figure 8. Setup for The Flippin' Pompoms. Variations of spheres are visible in the image. On the bottom left, the motorized device for automatic rotation of a ball is shown. On the top right of the screen, the image captured by the camera of the ball currently resting on the rotating device is shown. Below, also on the screen, the segmented colored areas of this sphere can be seen. Finally, the AOG formulas resulting from this segmentation can be seen on the left side of the screen. These can also be edited directly.

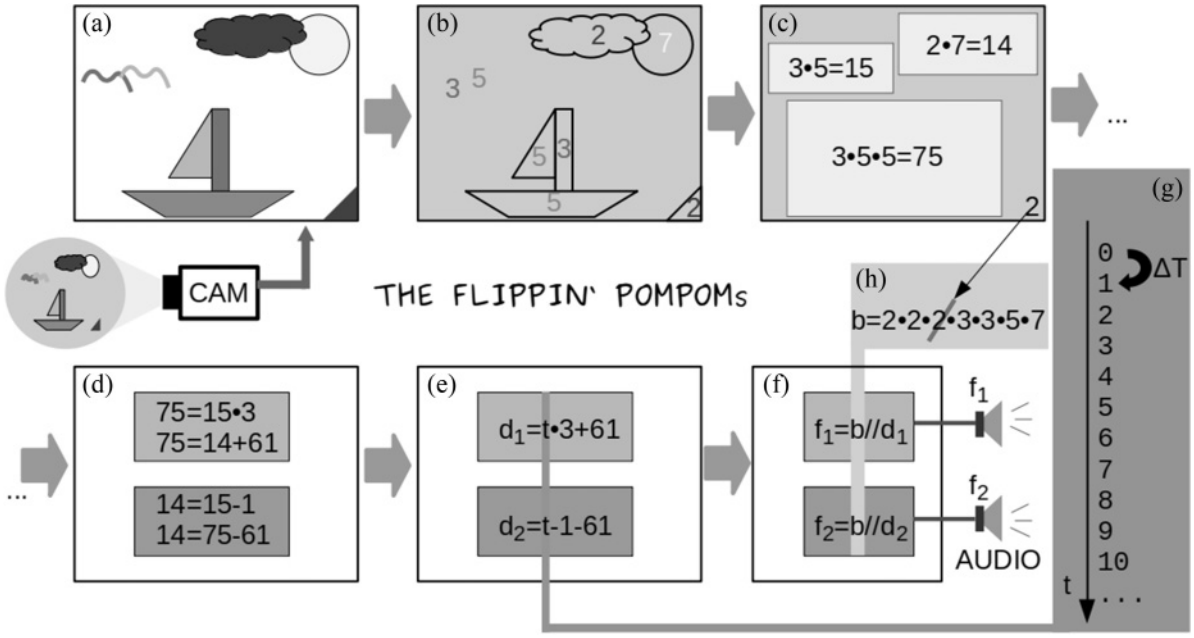


the number of areas in each color is counted. Using a color coding in which blue represents 2 red 3 green 5 and yellow 7 a multiplication of all the prime numbers assigned to the areas with each other is performed, and one obtains exactly one number for each individual contiguous colored area (Figures 9b, c).

Now the software determines which arithmetic operation is best suited to get from one number to another. The order in which the areas are selected is determined on the basis of neighborhood relations and the absolute sizes of the areas in pixels. When selecting the "suitable" operations, the system first looks to see whether a modulo operation is possible in which the operand c lies somewhere between the two values a and b. The relation between a, b, and c is then $a \bmod c = b$. The idea behind this is that only if $a > c > b$ can b be a real remainder of the modulo operation $a \bmod c$. For example, $100 \bmod 30 = 10$. If this fails, the software checks whether multiplication or division is possible. If this also fails, an addition or subtraction is always possible to get from one number to another (Figure 9d). Because the nearest neighbor is always taken from the largest areas, a path is created from a maximum of four largest areas, which encompasses different colored areas. The different paths can also run over the same areas to some extent. This has proven to be beneficial for the quality of the music, probably because the resulting voices have significant commonality while still differing slightly. This has not yet been examined in more detail. Each of the sequences of operations found then represents a formula in the sense of AOG. The further processing of these formulas, up to the conversion into sounds, is performed from this point on in the same way as with the formula editor, i.e., as described earlier in the section "Arithmetic Operation Grammar" and as visualized in Figures 9e–h.

In fact, the software for TFP is based directly on that of the formula editor. This goes so far that the latter is still displayed in TFP (as shown in Figure 8) and changes to the formulas can also be

Figure 9. Specific processing scheme of The Flippin' Pompoms from image capture to sound generation (see text).



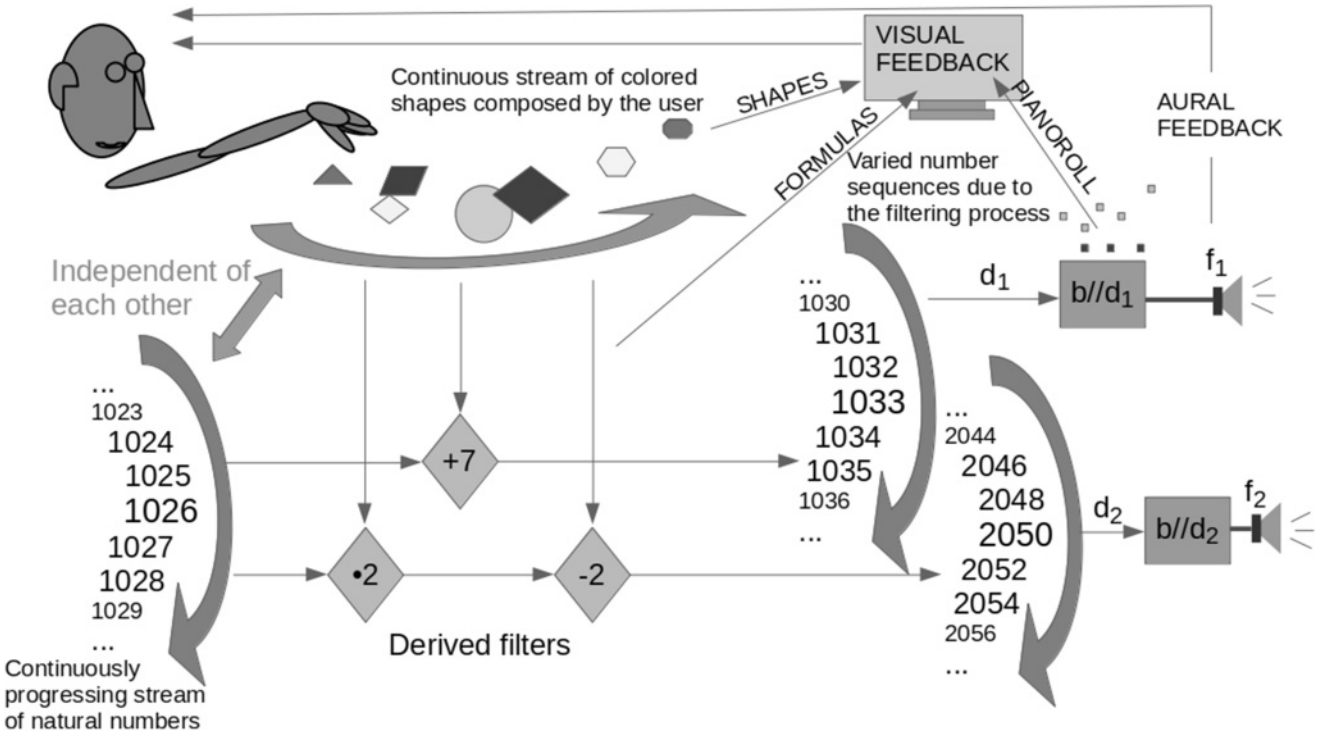
made there. Thus, a multimodal UI is available here, which provides possibilities for influencing the formulas on two possible levels, whereby the ball design always represents the entire composition and the currently visible formula typically represents a phrase that varies slightly over the course of time. In particular, however, TFP always keeps the AOG-related perspective open and thus offers users the opportunity to theoretically understand the underlying compositional principle and to emancipate themselves from the use of the given tool.

Discussion

To open up access to composing with AOG, especially for primary school children, without requiring knowledge of school arithmetic, advantage is taken of the fact that arithmetic operations ultimately help to handle sets. The route back to this starting point was taken by letting children arrange colorful forms into groups. In terms of data sonification, the children create a kind of data set. At first, I noticed that, because the compositions are realized with rotating spheres decorated with colored surfaces, the three-dimensionality in the 2-D camera image results in different topologies of the colored groups, depending on the perspective from which the webcam views the sphere. Although children are not working directly with musical elements, by applying this technique of recombining colored surfaces, something emerges that has much in common with composing. Varying musical structures in this way would be extremely difficult when working directly in a score. The decoration of the balls opens up a powerful and efficient metalevel with which children can compose. This is probably the deeper reason for the usefulness of the so-called facade pattern used in software development. In the search for descriptive possibilities to characterize meaningful settings for ubiquitous music, design patterns were also considered (Keller, Lazzarini, and Pimenta 2014, pp. xi–xxiii).

The attempt is now being made to transfer something from the world of design patterns, whose elements often characterize software components, to the overall system presented here, which includes not only the software but also the user and hardware. The facade pattern represents a structure that seems

Figure 10. Abstract processing scheme of The Flippin' Pompoms from image capture to sound generation (see text for details).



to correspond to the previously described setting: It simplifies the use of a subsystem consisting of many components by allowing a client to ensure the completion of common tasks via a single method instead of making a multitude of method calls to different elements of the subsystem to achieve the same goal. The nonstandard client still retains the ability to make calls at the subsystem level that are not bundled in the facade (Gemma et al. 1995, pp. 185–193). The role of the client is played here by TFP users. The balls to be decorated represent the facade, and the subsystem controlled by them is all the rest, consisting of the formula level down to the sound generation, as was seen in Figure 8. It is possible to bypass the facade by directly typing in AOG formulas and by getting feedback directly from the subsystem in the form of the perceptible musical event, its visualization as a piano roll, and the viewing of the AOG formulas (see Figure 10). This means that at the beginning of working with TFP, the facade completely hides the subsystem: The children tinker with balls, which they decorate with colored shapes. Little by little, via the feedback channels, the user (the client) gains a deeper knowledge of the connections behind the facade (in this case, the ball). Little by little, certain contexts are recognized, how certain configurations of the balls can influence the sound, as follows.

While a transparent sphere is spinning smoothly, and if the shape of the object inside is more cuboid rather than spherical, it can happen that the inside shape suddenly falls from one side to another. Such a change in the shape of an object leads to sudden changes. For this type of twisting, areas can be covered by others and new combinations of colored areas can emerge. Moving elements within the ball can gradually combine with other elements of the ball surface through the ball's rotation, and thus vary the sound event. The colors determine which instrument dominates. Isolated, single-color areas are used to influence the base number. There may be, for instance, areas with more- or less-complex patterns; see Figure 11. From the perspective of a person who has just started with TFP, all this

Figure 11. Decorating transparent and opaque spheres as a compositional method.

happens at the beginning, either by chance or by intentional reference to the visual form rather than the sonic one. In the best case, however, over time these relationships become increasingly clear and can be taken advantage of by the user. On the level of the facade design model, this would mean that instead of an initial, simplified method of accessing the functionality of the subsystem, methods of greater sophistication would gradually emerge, passing more parameters. Or these parameters have always been passed, but gradually the user becomes aware of them. The Flippin' Pompoms can be used to convey the meaning of what it is like to compose and to make the experience of holding one's own composition (as a ball) in one's hand and making it heard whenever desired.

Conclusions and Further Work

In this article, we explained in detail what Arithmetic Operation Grammar is and what potential it has in combination with suitable UIs to give laypeople the opportunity to compose their own music. But what contribution does this work make in the field of ubiquitous music and sonification? This should be made more explicit in this concluding section.

From the perspective of sonification, the method presented here differs from other sonification methods in that time does not have to be present as an order parameter in the input data, as would be the case with a still image. A time parameter—as would be present if the input were represented by a video—is in any case a prerequisite for sonifications that do not themselves have a temporal macrostructure (see, e.g., Braund and Miranda 2013; Denjean et al. 2019). Nor does the microstructure—that is, the actual sound—need to be obtained directly from input data, as is done in sonification methods that essentially compress or stretch the system time of the source data to obtain an audible frequency range (as by Holtzman et al. 2013). This temporal independency makes the AOG approach ideal for sonification of data that does not, in itself, contain time as an ordering parameter, and none of the other parameters need be interpreted as time. This does not, however, exclude the possibility of sonifying data in which time does appear as a parameter. Because of this independence, the AOG method also allows a free choice in the design of the sound and, thus, the possibility of using high-quality, software-based musical instruments. Finally, the resulting

macrostructure meets higher musical demands than might be met by many other common sonification methods (see e.g., Matsubara, Morimoto, and Uchide 2019). Overall, it can be said that there could be application niches for the procedure presented here in the field of sonification.

Because as described above, AOG is an example of a Chomskian third-order grammar, it is not necessary to explicitly acquire a theoretical apparatus of rules, even if at some point one emancipates oneself from the provided software tool and goes one's own way on the basis of AOG. Unlike comparable software tools, complexity is not hidden but the underlying mechanism is per se simple and easy to learn.

For example, Gil Weinberg (2002) discusses holding back the theoretical background. He justifies this by asserting that laypeople study differently than professionals, arguing that the underlying theory would overwhelm the target group and spoil the joy of composing. Furthermore, Weinberg states that such initial experiences can motivate the study of music theory at some later point. But that claim is formulated more as a hope than as a consequence of the use of the described composition tool.

> I believe that providing novices with the power to create and phrase a melody by manipulating its contour, regardless of its exact pitches and intervals, offers them a unique creative experience that is usually reserved for experts and that can serve as an entry point for further investigations into more advanced concepts such as harmony and counterpoint (Weinberg 2002, p. 45).

It would, of course, be possible at this point to counter the arguments made against the teaching of music theory to laypeople, such as the obvious fact that many professionals no longer need to study music theory at all and that laypeople are not necessarily associated with a lack of comprehension. On the other hand, one has to give credit to the approach taken by Weinberg, and to related approaches, for having successfully given novices the ability to also record musical ideas. This possibility is not available for users of AOG in its current form. The dominant experimenting with formulas and forms is not currently compatible with AOG. Instead, the two UIs presented here each provide a compact, easy-to-grasp representation layer of the emerging composition. In the one case it is the symbolic AOG formula, in the other it is the colorfully decorated spheres representing entire compositions. The audience experiment discussed earlier has also shown that it is precisely this compactness of the representation that greatly promotes creative cooperation between the people involved, because on this basis an exchange of ideas between the participants is strongly promoted. This shows a certain affinity to the tagging metaphor (Keller, Lazzarini, and Pimenta 2014, p. xviii). In connection with this metaphor, Keller and colleagues point out that creative activities take place through interaction with material or mental objects and that these are at best designed to form a suitable channel for these interactions.

On the whole, one can simply say that different approaches to helping laypeople to compose can each be successful in their own way, in the sense that "music is inclusive, and musics and their techniques and forms are cumulative, not mutually exclusive" (Spiegel 1998, p. 6).

But with respect to the work presented here, one could object that, for all the universality of the set of natural numbers $\mathbb{N}$, the special operations introduced and applied to it indirectly result in a preselection of the sound material provided, and that, furthermore, the musical examples presented all have a certain characteristic style that one may or may not like. At this point it is up to me now to express my hopes and convictions, which, however, with reference to work that has already been continued but not yet completed, can be presented in conclusion in a well-founded manner:

The decision to use $id(\mathbb{N})$ as the basic element was taken mainly because it was possible to prove properties described as musical. If one drops this restriction and goes over to the use of other types of potentially infinite series, completely different perspectives arise with regard to the results that can be achieved, as can be seen from the examples in Section 10 at http://kramann.info/98_AOG. As a preliminary justification for this step it may be

said that all these other series can be represented as mathematical mappings of the basic sequence $id(\mathbb{N})$. Whether it will be possible to extend the theory introduced here by such elements, while simultaneously preserving the current compact form, remains to be seen.

In the meantime, more extensive research has been conducted into the relationship between compositions generated with AOG and traditionally composed music, in which sound events with specific pitches are organized in time. A corresponding experiment can be seen at Section 8 on http://kramann.info/98_AOG. By means of an optimization algorithm, an attempt was made to optimize a group of four AOG formulas, each containing 16 operations, in such a way that for the length of four ticks at a certain starting value t_0 they represent the familiar canon "Frère Jacques," if all four voices have already begun there. This was successful, but in the ticks before, the musical set obtained with AOG converges towards the desired piece and immediately afterwards diverges away from it. This is straightforward to follow in the included sound conversion. As a preliminary result, a solution of this inverse problem may be possible in principle, but is by no means trivial. To achieve better results, the next step would be to test alternative operations.

A criticism may also be raised that the UIs presented in this article, which are more oriented towards pictorial design, do not convey the satisfaction of experiencing a direct sonic reaction resulting from an action. Feedback mechanisms, however, which have the goal of tracing such sound events directly generated by the user back to AOG formulas that could have generated them to obtain automatic accompaniment, are currently also the subject of further development of the possibilities that arise with AOG (see Section 7 at http://kramann.info/98_AOG for samples). The advantage of the offline approaches, where a composition is described but not played, is that they provide the possibility of being embraced, distributed, and handled cooperatively by participants who are far away from each other, since latencies do not play a major role. In particular, automatic and parallel sound conversion can also take place asynchronously at the respective locations. This possibility is also mentioned by Lazzarini et al. (2014) for the design of ubiquitous music systems.

Last but not least, AOG's explicitly comprehensible connection between music and mathematics offers welcome starting points for the planned STEM project day and all those who may follow it, to turn it into a STEAM project day (with an "A" for the arts) in the sense of John Maeda (2013), thus breaking the ground for a way of thinking that goes beyond the well-worn categories of isolated disciplines and making a significant contribution to bringing forward the "every day creativity" (Keller, Lazzarini, and Pimenta 2014, pp. 5–7, 29–30) in the field of ubiquitous music.

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