

Life Worth Mentioning: Complexity in Life-Like Cellular Automata

Abstract Cellular automata (CA) have been lauded for their ability to generate complex global patterns from simple local rules. The late English mathematician, John Horton Conway, developed his illustrious *Game of Life* (Life) CA in 1970, which has since remained one of the most quintessential CA constructions—capable of producing a myriad of complex dynamic patterns and computational universality. Life and several other Life-like rules have been classified in the same group of aesthetically and dynamically interesting CA rules characterized by their complex behaviors. However, a rigorous quantitative comparison among similarly classified Life-like rules has not yet been fully established. Here we show that Life is capable of maintaining as much complexity as similar rules while remaining the most parsimonious. In other words, Life contains a consistent amount of complexity throughout its evolution, with the least number of rule conditions compared to other Life-like rules. We also found that the complexity of higher density Life-like rules, which themselves contain the Life rule as a subset, form a distinct concave density-complexity relationship whereby an optimal complexity candidate is proposed. Our results also support the notion that Life functions as the basic ingredient for cultivating the balance between structure and randomness to maintain complexity in 2D CA for low- and high-density regimes, especially over many iterations. This work highlights the genius of John Horton Conway and serves as a testament to his timeless marvel, which is referred to simply as: Life.

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I Introduction

Cellular automata (CA) are discrete dynamical systems used primarily as a computational model to study how spatio-temporal patterns evolve in a wide range of phenomena. The model encompasses a grid of cells, each of which takes a discrete state. The dynamics of the state of a cell is governed by transition rules which involve the state of the cell itself along with the states of its surrounding neighbors. There are many cases in which CA follow simple rules yet lead to interesting complex patterns (Wolfram, 1982, 2002; Wolfram & Packard, 1985).

One such rule that has gained the most popularity by far is Conway's *Game of Life* (Life) (Berlekamp et al., 1982; Gardner, 1970). It was in 1970 when John Horton Conway, after experimenting with

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various CA rules, settled on Life, which held analogies to real living organisms—forming parallels to birth, death, and survival in the natural world. Live cells (i.e., cells whose state is 1) with too few live neighbors experience death from isolation, whereas those with too many live neighbors experience death from suffocation. The rule in its entirety is as follows: (i) a dead cell (i.e., a cell whose state is 0) that is surrounded by exactly three live neighbors will be born into a live cell; (ii) a live cell with exactly two or three live neighbors will survive; and (iii) otherwise, all other cells will die or remain dead. The study of Life abounds, including self-organizing criticality, autopoiesis, structural patterns, interesting initial conditions, generated components as information-carrying agents, and computational ability (Bak et al., 1989; Beer, 2004; Gotts, 2000; Gutowitz et al., 1987; Lizier et al., 2014; Rendell, 2002; Turney, 2021a). Life has found its way into a myriad of disciplines such as mathematics, physics, computer science, information theory, biology, ecology, art, and even music (Axelrod, 1997; Miranda & Kirke, 2010).

Understanding how Life compares to other two-dimensional CA was attempted by Wolfram (1984), in which Life was assigned to Class IV for CA rules that produce complex, unpredictable behavior. Other works have argued for more objective means of classification that are better able to distinguish intrinsic complexity among CA rules (Baetens & De Baets, 2010; Borriello & Walker, 2017; Li et al., 1990; Turney, 2021b; Wuensche, 1999; Zenil & Villarreal-Zapata, 2013), or that show to what extent the feat itself is undecidable (Culik & Yu, 1988). Moreover, Life has inspired an entire category of CA rules referred to as *Life-like* rules (Catagolue, 2021; LifeWiki, 2021)—rules that behave similarly to Life and that are likely all in Class IV. The behavior of many Life-like rules has been individually studied in great detail (Eppstein, 2010; Johnston, 2010). Although Life-like rules are collectively considered to exhibit complex, interesting behavior overall, a rigorous quantitative comparison between Life and other Life-like rules has not been fully established.

Here we show that Life naturally tends toward lower density and higher information content while remaining the most parsimonious among popular Life-like rules. In other words, Life is capable of maintaining a consistent amount of complexity throughout its evolution after transience with the least number of rule conditions compared to other Life-like rules. We also show that the complexity of higher density Life-like rules, which themselves contain Life, form a natural density-complexity relationship. Through this analysis, we show why the Life rule constructed by John Horton Conway may very well be the most interesting and fundamental CA construction with regards to its complexity and its role in how complexity is manifested in Life-like rules more generally.

2 Material and Methods

The complexity intrinsic to Life and Life-like rules is possible through the balance of order and disorder. We demonstrate this by simulating Life and collecting density and complexity measurements of the resulting dynamic patterns. Density is defined as the average ratio of the number of cells whose state is 1 to the number of total cells on a grid ($\rho = |S|_1/n$). To ensure that the generated patterns were indeed characterized by Life, random initial conditions with varying densities ($\rho = 0.1\text{--}0.5$) were simulated for a sufficiently long period of time (30–100 iterations) before measuring the complexity. This was performed for four different grid sizes: 20×20 , 30×30 , 40×40 , and 50×50 . With each grid size, 5,000 final Life patterns were generated whereby complexity data were averaged at each density value.

Life was then compared to other Life-like rules in terms of the long-term dynamics of their density and complexity. The B_x/S_y naming convention is used to refer to various rules for 2D outer-totalistic CA (Eppstein, 2010). The B stands for *birth* and the S stands for *survival*. The x and y are subsets that can contain digits from 0 to 8 which represent the number of live neighboring cells in a 3-by-3 Moore neighborhood. This B_x and S_y corresponds to giving birth to a cell from being dead and allowing a live cell to survive, respectively. The Life-like rules discussed here naturally decrease density from random initial conditions yet continue dynamic activity for an extended period. In other words, these rules neither quickly stabilize a grid to low densities nor force grids to explode in density.

The complexity produced by these rules is compared in two phases. First, we choose known rules that most closely follow Life's trajectory in the density-complexity domain. These rules include: B3/S238 (EightLife), B38/S238 (HoneyLife), B368/S238 (LowDeath), B38/S23 (PedestrianLife), B36/S125 (2×2), B36/S23 (HighLife), and B3/S12 (FlockLife). These are well-known rules in the CA community for behaving most similarly to Life (Catagolue, 2021; Eppstein, 2010; Johnston, 2010; LifeWiki, 2021). Second, we explore lesser known Life-like rules that allow higher density configurations after transience. These higher density rules, like many of the previously mentioned rules, contain the parsimonious Life rule within them as a subset (i.e., B3.../S23...). The higher density candidate rules chosen for this include: B37/S23 (DryLife), B367/S23 (DrighLife), B3578/S238, B3578/S23, B356/S23, B356/S238, B3568/S23, B3568/S238, B3578/S237, and B35/S236. The unnamed rules are the ones we discovered ourselves through computational exploration of the Life-like rule space. For both phases of rule comparison, grids are randomly initialized with 50% density on a 50×50 cell grid and evolved for 2,000 time steps. Density and complexity values are averaged over 50 experimental runs at each time step throughout the temporal evolution. Toroidal (periodic) boundary conditions were enforced for all CA simulations.

2.1 Conditional Entropy and Complexity Measurement

Conditional entropy has been used to characterize spatial correlations and aesthetic structure in 2D CA (Andrienko et al., 2000; Bates & Shepard, 1993; Wackerbauer et al., 1994). (This measure was originally termed *information gain complexity*, but we use the term *conditional entropy* to avoid misleading nomenclature.) Generally, conditional entropy, $H(Y|X)$, measures the amount of information needed to describe a random variable Y given knowledge of the value of a random variable X :

$$H(Y|X) = - \sum_{y,x} P(y,x) \log_2 P(y|x) \quad (1)$$

where y and x are the discrete states of Y and X , $P(y,x)$ is the joint probability of y and x , and $P(y|x)$ is the conditional probability of y given x . Conditional entropy may also be defined as the average remaining uncertainty in Y after observations of X are made. In the context of CA, conditional entropy has been used to measure how much information is needed on average to determine the state of a cell in the 2D CA given that its neighboring cell states are known. Consider the state variable pairs of all cells and their upward neighbors, (S, U) , on a CA grid. The conditional entropy can be calculated as follows:

$$H(S|U) = - \sum_{s,u} P(s,u) \log_2 P(s|u) \quad (2)$$

where s and u are the discrete states (i.e., $\{0, 1\}$) of the cell state variables S and U . The conditional entropy for a given cell is calculated using the four von Neumann neighborhood directions (i.e., up, down, left, right, denoted by states variables U, D, L, R). These four directional conditional entropy values are then averaged to assign a single value Γ to a given 2D configuration which captures the structural complexity of the CA pattern:

$$\Gamma = \frac{1}{4} [H(S|U) + H(S|D) + H(S|L) + H(S|R)]. \quad (3)$$

To quantify the extent to which individual cells are interdependent with each other, Γ is used to compare Life configurations to corresponding randomized configurations with equal live cell density. The randomized configuration serves as an objective control with which other Life-like rules can be compared since they each exhibit unique cell densities as they evolve. This may also be understood analytically as the unconditional Γ or simply the Shannon entropy: $H(S) = - \sum_s P(s) \log_2 P(s)$ where s is

the cell state (i.e., $\{0, 1\}$). We use the difference in Γ between Life (or any Life-like CA rule) and its randomized configuration counterpart as an information-theoretic measure of how much non-random structure emerges in the CA grid:

$$\Delta = \Gamma_{\text{Random}} - \Gamma_{\text{Rule}} = H(S) - \Gamma_{\text{Rule}}. \tag{4}$$

Note that $\Delta \geq 0$ since Γ is maximal when the configuration is entirely random. The greater the value of Δ is, the further away from randomness a particular CA configuration is understood to be. In other words, Δ is a complexity measure which quantifies the extent to which adjacent cells show interdependence.

3 Results

3.1 Complexity of Life Configurations

Figure 1 shows the measured Γ for both random configurations and evolved Life configurations (Figure 1(a)) and the proposed complexity measure Δ (Figure 1(b)) for four different grid sizes: 20×20 , 30×30 , 40×40 , and 50×50 . For each grid size, 5,000 final Life patterns were generated whereby Γ and Δ were averaged for each unique density value. Density in this case is defined as the number of live cells on a grid ($|S|_1 = \rho n$). The Δ measure remains consistently maximized with a higher number of live cells, at around 0.04 bits for Life (Figure 1(b)). The convergence toward this maximal value strongly indicates that Δ is a measure of the rule’s inherent complexity independent of grid size. This scale-independent property of the rule suggests that despite the details of local structural patterns, Life robustly maintains a level of balance between order and randomness.

3.2 Parsimony of Life Rules

Figure 2 shows the evolution of the average density and complexity of Life-like patterns settling to distinct values over time with dynamics governed by each rule. The FlockLife and HighLife rules maintain low density patterns but also cause these patterns to decay so much as to become stable and unable to maintain dynamic activity for long periods of time compared to Life. The 2×2 rule (Johnston, 2010) produces higher density patterns whose complexity is low with respect to Life. The rules that most closely follow Life’s density-complexity trajectory through time are EightLife, HoneyLife, LowDeath, and PedestrianLife. The 95% confidence interval in Figure 2 was constructed with 50 simulations for each rule evolved over 2,000 time steps and demonstrates the similarity in dynamics among these rules. Moreover, these rules which exhibit similar density-complexity trajectories to Life contain the Life rule (i.e., $B3/S23$) as a subset of their conditions. Hence, the Life rule itself

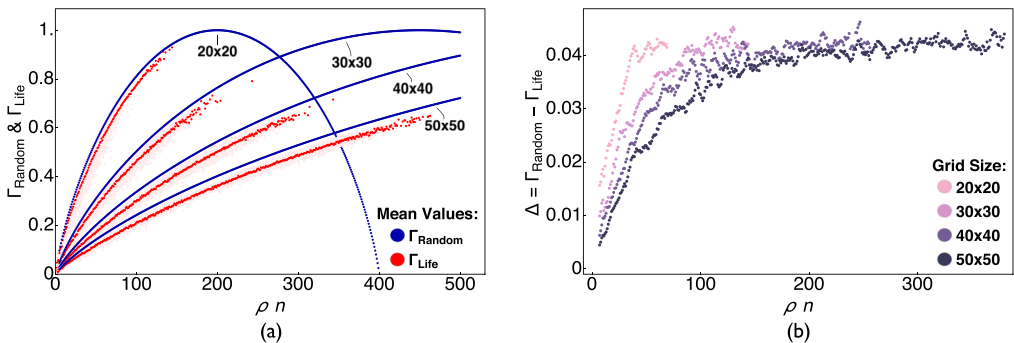


Figure 1. Game of life density and complexity measure. (a) Life (red) and random (blue) configuration Γ values for a range of densities defined by the number of live cells on a grid (ρn). (b) Complexity measure Δ converges to a value at about 0.04 bits as the number of live cells increases and remains independent of grid size.

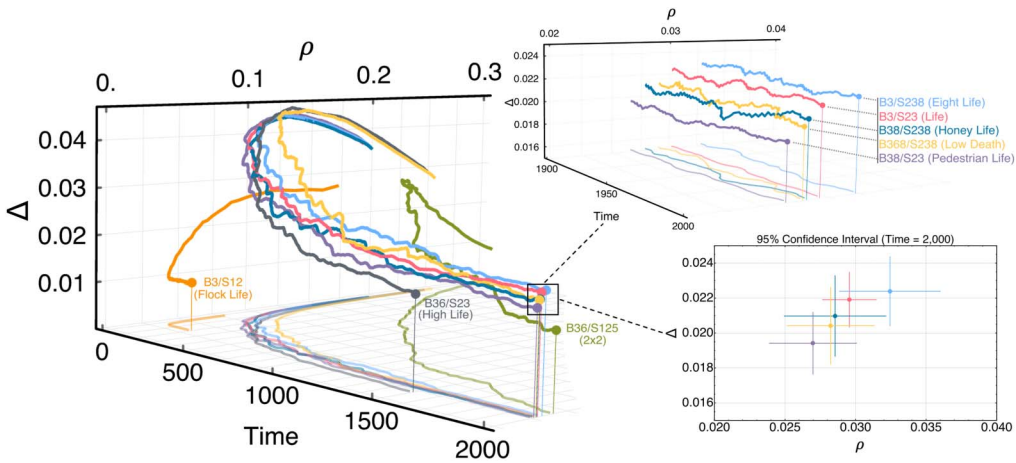


Figure 2. Temporal evolution of low density Life-like rules whereby density (ρ) and complexity (Δ) are measured and averaged for each point in time over 2,000 time steps and 50 simulations for each rule.

contains the least number of conditions making it the most parsimonious rule capable of exhibiting this level of complexity and density. This indicates that John Horton Conway successfully constructed an optimal rule that not only maintains complex activity over many evolutions but also contains the smallest number of needed conditions to manifest this behavior. Moreover, the fact that each rule contains Life itself suggests that Life is fundamental in constructing complex dynamic patterns. Higher complexity may be achieved for rules whose temporal evolution leads to higher densities after transient activity. Not only is Life unique in its ability to spawn low density patterns which give rise to long-lasting complex behavior, but it is also fundamental and a flexible elementary constituent for complex Life-like behavior to emerge for higher density patterns.

3.3 Searching for Higher Density Life-Like Rules

Figure 3 shows the temporal evolution of rules that maintain higher density patterns and contain the Life rule within them (i.e., B3.../S23...) and that were selected after manual exploration of the rule space. Life itself is also included in this list of rules for convenient comparison. As the rules evolve

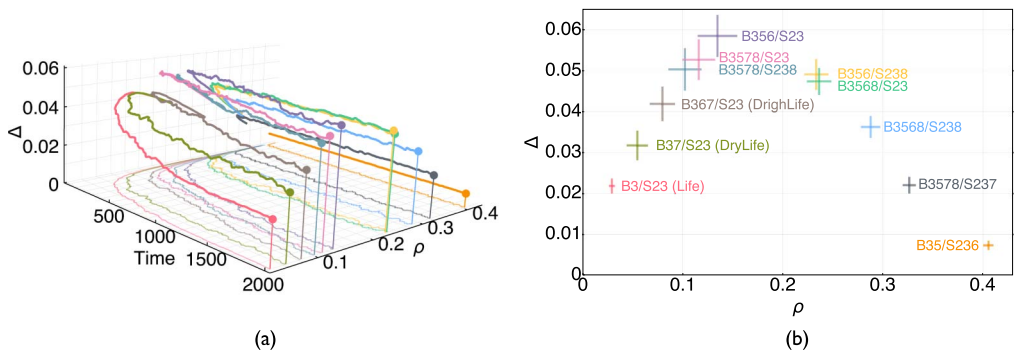


Figure 3. Temporal evolution of high density Life-like rules. (a) Temporal evolution of rules that maintain higher density patterns and contain the Life rule within them (i.e., B3.../S23...). (b) Density (ρ) and complexity (Δ) measurements with 95% confidence intervals after transient activity (i.e., Time = 2,000) for higher density Life-like rule candidates.

through time, their complexity values settle to distinct regions in the density-complexity domain (Figure 3(a)). A density-complexity concave relationship is revealed among the candidate Life-like rules after transient behavior from random initial conditions (Figure 3(b)). As the density increases after transience, the complexity increases as well until the density becomes too high for complex patterns to be possible. At this point, the grid saturates with live cells and produces highly random and structureless activity thereby decreasing the complexity of the pattern. This relationship also indicates that there is likely a critical point between Life and pure randomness which maximizes complexity in 2D CA. In this context, optimal complexity is characterized in terms of structural aesthetics and information-richness that occurs for a rule after transient activity. We find that a possible optimal candidate is rule B356/S23, which we have named *Dynamic Life*.

4 Conclusion

In this article, we have constructed a measure of complexity for 2D CA patterns. This measure incorporates the inherent aesthetic structure and affinity to randomness a pattern contains. We used this measure to investigate the complexity produced by Conway's Game of Life and showed that complexity is a scale-independent property of the rule. We compared Life to similar rules with respect to their space-filling and complexity characteristics through time. We found that Life can maintain similar long-lasting, complex behavior like other well-known Life-like rules throughout its temporal evolution and with the least number of conditions (i.e., B3/S23). This indicates that Life is likely the most parsimonious rule capable of producing this level of density and robust complexity. Finally, we investigated the temporal evolutions of higher density Life-like rules that contain the Life rule as a subset of their condition (i.e., B3.../S23...). A concave relationship was uncovered in the density-complexity domain after initial transient activity. This indicates a likely optimal rule (i.e., B356/S23) between Life and pure randomness which maximizes complexity in 2D CA. Moreover, since Life is used as the key ingredient to construct complex and Life-like patterns of a wide range of density and complexity values, Life appears to serve as an elementary constituent responsible for complex Life-like behavior to emerge in both lower and higher density configurations. Future research directions include measuring spatio-temporal information transfers in Life-like patterns to investigate long-time and long-distance correlations (Lizier et al., 2014). The work presented here further sheds light on why John Horton Conway's Game of Life is considered the canonical Life-like rule and continues to be studied by students and academics even today—certainly making it a *life worth mentioning*.

Acknowledgments

We would like to recognize and honor the work and life of the late mathematician John Horton Conway—the creator of the Game of Life among many other major contributions in the field of mathematics and beyond.

Author Contributions

Eric Peña and Hiroki Sayama conceived the research idea and developed the complexity measure. Eric Peña wrote the code and carried out the experiments and simulations. Eric Peña took the lead in writing the manuscript. Hiroki Sayama supervised the work, provided critical feedback, and helped shape the research and manuscript.

Competing Interests

The authors declare no competing interests.

Data Availability

The data sets generated and analyzed for this study are available from the corresponding author on reasonable request.

Code Availability

The code used to compile and analyze data was written in Python 3.7.9. The code to replicate analysis along with simulations of the rules mentioned in this article are available from the corresponding author on reasonable request.

References

- Andrienko, Y. A., Brilliantov, N. V., & Kurths, J. (2000). Complexity of two-dimensional patterns. *European Physical Journal B*, 15(3), 539–546. <https://doi.org/10.1007/s100510051157>
- Axelrod, R. (1997). Advancing the art of simulation in the social sciences. In R. Conte, R. Hagselmann, & P. Terna (Eds.), *Simulating social phenomena* (pp. 21–40). Springer. https://doi.org/10.1007/978-3-662-03366-1_2
- Baetens, J. M., & De Baets, B. (2010). Phenomenological study of irregular cellular automata based on Lyapunov exponents and Jacobians. *Chaos*, 20(3), Article 033112. <https://doi.org/10.1063/1.3460362>
- Bak, P., Chen, K., & M., C. (1989). Self-organized criticality in the Game of Life. *Nature*, 342(6251), 780–782. <https://doi.org/10.1038/342780a0>
- Bates, J. E., & Shepard, H. K. (1993). Measuring complexity using information fluctuation. *Modern Physics Letters A*, 172(6), 416–425. [https://doi.org/10.1016/0375-9601\(93\)90232-O](https://doi.org/10.1016/0375-9601(93)90232-O)
- Beer, R. D. (2004). Autopoiesis and cognition in the Game of Life. *Artificial Life*, 10(3), 309–326. <https://doi.org/10.1162/1064546041255539>, PubMed: 15245630
- Berlekamp, E. R., Conway, J. H., & Guy, R. K. (1982). *Winning ways for your mathematical plays* (Vol. 4). CRC Press.
- Borriello, E., & Imari Walker, S. (2017). An information-based classification of elementary cellular automata. *Complexity*, 2017, Article 1280351. <https://doi.org/10.1155/2017/1280351>
- Catagolue. (2021, March). *Outer-totalistic rules*. <https://catagolue.hatsya.com/rules/lifelike>
- Culik II, K., & Yu, S. (1988). Undecidability of CA classification schemes. *Complex Systems*, 2(2), 177–190.
- Eppstein, D. (2010). Growth and decay in Life-like cellular automata. In A. Adamatzky (Ed.), *Game of Life cellular automata* (pp. 71–97). Springer. https://doi.org/10.1007/978-1-84996-217-9_6
- Gardner, M. (1970). Mathematical games: The fantastic combinations of John Conway’s new solitaire game “Life.” *Scientific American*, 223(4), 120–123. <https://doi.org/10.1038/scientificamerican1070-120>
- Gotts, N. M. (2000). Emergent phenomena in large sparse random arrays of Conway’s Game of Life. *International Journal of Systems Science*, 31(7), 873–894. <https://doi.org/10.1080/002077200406598>
- Gutowitz, H. A., Victor, J. D., & Knight, B. W. (1987). Local structure theory for cellular automata. *Physica D*, 28(1–2), 18–48. [https://doi.org/10.1016/0167-2789\(87\)90120-5](https://doi.org/10.1016/0167-2789(87)90120-5)
- Johnston, N. (2010). The B36/S125 “2x2” Life-like cellular automaton. In A. Adamatzky (Ed.), *Game of Life cellular automata* (pp. 99–114). Springer. https://doi.org/10.1007/978-1-84996-217-9_7
- Li, W., Packard, N. H., & Langton, C. G. (1990). Transition phenomena in cellular automata rule space. *Physica D*, 45(1–3), 77–94. [https://doi.org/10.1016/0167-2789\(90\)90175-O](https://doi.org/10.1016/0167-2789(90)90175-O)
- LifeWiki. (2021, March). *List of Life-like cellular automata*. https://www.conwaylife.com/wiki/List_of_Life-like_cellular_automata
- Lizier, J. T., Prokopenko, M., & Zomaya, A. Y. (2014). A framework for the local information dynamics of distributed computation in complex systems. In M. Prokopenko (Ed.), *Guided self-organization: Inception* (pp. 115–158). Springer. https://doi.org/10.1007/978-3-642-53734-9_5
- Miranda, E. R., & Kirke, A. (2010). Game of Life music. In A. Adamatzky (Ed.), *Game of Life cellular automata* (pp. 489–501). Springer. https://doi.org/10.1007/978-1-84996-217-9_24

- Rendell, P. (2002). Turing Universality of the Game of Life. In A. Adamatzky (Ed.), *Collision-based computing* (pp. 513–539). Springer. https://doi.org/10.1007/978-1-4471-0129-1_18
- Turney, P. D. (2021a). Evolution of autopoiesis and multicellularity in the Game of Life. *Artificial Life*, 27(1), 26–43. https://doi.org/10.1162/artl_a_00334
- Turney, P. D. (2021b). Measuring behavioral similarity of cellular automata. *Artificial Life*, 27(1), 62–71. https://doi.org/10.1162/artl_a_00337
- Wackerbauer, R., Witt, A., Atmanspacher, H., Kurths, J., & Scheingraber, H. (1994). A comparative classification of complexity measures. *Chaos Solitons & Fractals*, 4(1), 133–173. [https://doi.org/10.1016/0960-0779\(94\)90023-X](https://doi.org/10.1016/0960-0779(94)90023-X)
- Wolfram, S. (1982). Statistical mechanics of cellular automata. *Reviews of Modern Physics*, 55(3), Article 601.
- Wolfram, S. (1984). Universality and complexity in cellular automata. *Physica D*, 10(1–2), 1–35. [https://doi.org/10.1016/0167-2789\(84\)90245-8](https://doi.org/10.1016/0167-2789(84)90245-8)
- Wolfram, S. (2002). *A new kind of science*. Wolfram Media.
- Wolfram, S., & Packard, N. H. (1985). Two-dimensional cellular automata. *Journal of Statistical Physics*, 38(5), 901–946.
- Wuensche, A. (1999). Classifying cellular automata automatically: Finding gliders, filtering, and relating space-time patterns, attractor basins, and the Z parameter. *Complexity*, 4(3), 47–66. [https://doi.org/10.1002/\(SICI\)1099-0526\(199901/02\)4:3<47::AID-CPLX9>3.0.CO;2-V](https://doi.org/10.1002/(SICI)1099-0526(199901/02)4:3<47::AID-CPLX9>3.0.CO;2-V)
- Zenil, H., & Villarreal-Zapata, E. (2013). Asymptotic behavior and ratios of complexity in cellular automata. *International Journal of Bifurcation and Chaos*, 23(9), Article 1350159. <https://doi.org/10.1142/S0218127413501599>