# Local-Global Interaction and the Emergence of Scale-Free Networks with Community Structures

Jing Liu**
University of New South Wales

Hussein A. Abbass**
University of New South Wales

Weicai Zhong*,**
University of New South Wales

David G. Green†
Monash University

**Abstract** Understanding complex networks in the real world is a nontrivial task. In the study of community structures we normally encounter several examples of these networks, which makes any statistical inferencing a challenging endeavor. Researchers resort to computer-generated networks that resemble networks encountered in the real world as a means to generate many networks with different sizes, while maintaining the real-world characteristics of interest. The generation of networks that resemble the real world turns out in itself to be a complex search problem. We present a new rewiring algorithm for the generation of networks with unique characteristics that combine the scale-free effects and community structures encountered in the real world. The algorithm is inspired by social interactions in the real world, whereby people tend to connect locally while occasionally they connect globally. This local-global coupling turns out to be a powerful characteristics that is required for our proposed rewiring algorithm to generate networks with community structures, power law distributions both in degree and in community size, positive assortative mixing by degree, and the rich-club phenomenon.

**Keywords**
Complex networks, community, scale-free networks



## 1 Introduction

Complex networks describe a wide range of systems in nature and society. Commonly cited examples include the Internet, e-mail interactions, gene regulatory networks, science collaboration networks, sensor and communication networks, citation networks, food webs, metabolic networks, and many more [2, 20, 29, 30, 39, 47, 52, 54]. Because of their significant contribution to our understanding of complex systems, complex networks have been attracting much interest and their study has undergone a remarkable development over the last decade. Most of the work in this field has concentrated on two aspects: properties found in real-world networks, and ways to find models to build understanding about the emergence of these properties and the self-organizing process in complex systems [19, 51].

Two of the best-known properties exhibited by many real-world networks are the small-world property [59] and the scale-free property [4]. The former suggests that a network has a high degree

* Contact author.
** School of Engineering and Information Technology, The University of New South Wales at the Australian Defence Force Academy, Canberra, ACT 2600, Australia. E-mail: jing.liu@adfa.edu.au (J.L.); h.abbass@adfa.edu.au (H.A.A.); w.zhong@adfa.edu.au (W.Z.)
† Centre for Research in Intelligent Systems, Monash University, Clayton, Victoria 3800, Australia. E-mail: david.green@monash.edu

 

of clustering and a small average distance between any two nodes, while the latter entails a power-law distribution of node degrees, $P(k) \sim k^{-\gamma}$, where $P(k)$ is the probability that a node in the network has $k$ connections to other nodes and $\gamma$ is a positive real number determined by the given network. With the progress of our understanding about networks, another property appeared on the stage and attracted considerable attention recently, namely, *communities* [21, 35, 41], which are groups of nodes characterized by having more internal than external connections between them.

There is a long list of work contributing to the understanding of the emergence of the above properties. The available network models are all based on one aspect of reality. Many scientists have focused their attention on growing networks in which a new node is added to networks over time [4]. However, as indicated by [31], growth models of this type are quite inappropriate as models of the growth of social networks. One of the reasons is that, although new nodes are of course continually added to social networks, the time scale on which people make and break social connections is much shorter than the time scale on which nodes join or leave the network. Thus, the addition and removal of nodes will not be a major factor determining the instantaneous structure of social networks, and to a first approximation these networks can therefore be treated using a model with a constant number of nodes but a varying number and arrangement of edges. Many models of this type have been proposed [24, 31, 33, 55], in which the frequently used methods to tune the connections are limited degree and random weights.

Further, there is also another important class of network model in which both the numbers of nodes and edges are constant through the whole evolutionary process, namely, rewiring network models. The classic example of Watts and Strogatz [59] for small-world networks is of this type, and there is also some work dedicated to the scale-free case [9, 42, 46, 61]. Network rewiring is related to some multiurn models [22, 23, 43, 44] and to the zero-range process [16, 17, 53]. Because most systems in the real world cannot grow indefinitely, networks of constant size have many applications, such as pottery designs, dog breeds, and baby name popularity in the transmission of cultural artifacts [5–7, 27, 28, 36], the distribution of family names in constant populations [62], the diversity of genes [12, 32], the voter model [34, 56], and minority game strategies [3]. Exact solutions are also given in [18] for the rewiring model of a bipartite graph, using a mixture of random and preferential attachment. The full mean-field equations for the degree distribution and its generating function are given.

Grönlund and Holme [25] extended the original seceder model [13, 14, 57] to a social network rewiring model by starting from an Erdős-Rényi (ER) random graph [15]. This model reproduces the emergence of community structures, which is attributed to the effect of the agents' personal rationales as well as to the properties of high clustering and positive assortative mixing by degree. However, the degree distribution in networks generated by this model is not a power law, but a peak around the average degree that is exponentially decaying. Moreover, no analysis of the distribution of community size is given.

In this article, we propose a rewiring network model based on social interactions. Although social interactions can be formalized in many different ways, one of the frequently used classifications being friendship [55], we use a more general and simpler way, namely, daily people-to-people interactions without considering their friendship. Such interactions between people often exhibit two distinct forms and occur at different times. In many scenarios, people normally interact with a small group of individuals, such as their office colleagues or close family members, but at times will come into contact with a wider group, as when they attend meetings or visit family members far away. We refer to the former case as *local interaction* and to the latter as *global interaction*. Here, we use the edge rewiring process in undirected graphs to depict such interactions.

Similar to the model in [25], in our model no weights are attached to edges and no limitation is put on the degree of each node. Also, the evolution starts from an ER graph, and only rewiring is conducted. However, on tuning the single parameter governing the ratio of the two types of interactions, the resulting networks undergo a gradual structural transition from a community-free and Gaussian degree distribution topology to one of scale-free degree distribution and community structure with both high quality and power law community size distribution.

In the rest of this article, the network model is first described in the next section. Sections 3 and 4 study the community structure and degree distribution of the generated networks, respectively. Section 5 studies assortative mixing at both the network level and local level. After that, Section 6 compares networks generated by our model with real-world Internet topology at the autonomous system (AS) level—namely, the AS graph—and studies the rich-club phenomenon. Finally, we discuss the experimental results and summarize the conclusions in Section 7.

## 2 Model

During the evolutionary process, a sequence of graphs $\{G_t\}$ is produced. The set of nodes, $V$, of each graph in this sequence is the same; it consists of $N$ nodes, while the set of edges, $E_t$, is time-dependent but always has $M$ edges. Here, the number of iterations of the algorithm determines the simulation time $t = 0, 1, 2, \ldots, t_{\max}$. $\{G_0\}$ is the initial graph, which is constructed by the ER random graph model: A graph with $N$ nodes and $M$ edges is constructed by starting from isolated nodes and then iteratively introducing edges between node pairs chosen with uniform randomness and with the restriction that no multiple edges or loops are allowed. However, since the original ER model does not guarantee the connectivity of the generated networks, we revise it slightly: Edges are first created to connect all nodes, and then more edges are created. After the initial graph is constructed, the following rewiring procedures are repeated until $t_{\max}$ is reached.

- *Step 1:* A node $v_i$ is selected with probability $P_{v_i}$, which is proportional to $v_i$'s degree $k_{v_i}$. That is,

$$P_{v_i} = \frac{k_{v_i}}{\sum_{j=1}^{N} k_{v_j}} \tag{1}$$

  Let $\mathbf{\Omega}_{v_i}$ be the set of nearest neighbors of $v_i$ (i.e., there is an edge connecting $v_i$ and each node in $\mathbf{\Omega}_{v_i}$), $\mathbf{\Omega}_{v_i}^{>1}$ be the set of nearest neighbors of $v_i$ whose degrees are larger than 1 (i.e., for each node in $\mathbf{\Omega}_{v_i}^{>1}$, $v_i$ is not the only nearest neighbor of that node), and $\mathbf{\Omega}_{v_i}^{2}$ be the set of nearest neighbors of all nodes in $\mathbf{\Omega}_{v_i}$.

- *Step 2:* Randomly draw a real number from [0, 1], and if this number is smaller than $P_{\text{local}}$, a predefined parameter, go to step 3; otherwise, go to step 4.
- *Step 3 (local interactions):*

  (i) If $\mathbf{\Omega}_{v_i}^{>1}$ or $\mathbf{\Omega}_{v_i}^{2}/\mathbf{\Omega}_{v_i}$ is empty, then go to step 1.

  (ii) Delete the edge between $v_i$ and $v_i^j$, where $v_i^j$ denotes the $j$th element in $\mathbf{\Omega}_{v}^{>1}$ and

$$\frac{\left|\mathbf{\Omega}_{v_i} \cap \mathbf{\Omega}_{v_i^j}\right|}{k_{v_i^j} - 1} \tag{2}$$

  is the smallest for $j = 1, 2, \ldots, |\mathbf{\Omega}_{v_i}^{>1}|$, where $|\cdot|$ denotes the cardinality of the set.

  (iii) If the resulting network is not connected, recover the edge deleted in (ii), and then go back to step 1.

  (iv) Select a node in $\mathbf{\Omega}_{v_i}^{2}$ based on a probability proportional to its degree to connect to $v_i$, and go to step 1.

- *Step 4 (global interactions):*

  (i) If $k_{v_i} = N - 1$, so that $v_i$ already connects to all other $N - 1$ nodes, go to step 1.

  (ii) Delete the edge between $v_i$ and one of its nearest neighbors $v_i^j$, and select $v_i^j$ based on a probability inversely proportional to its degree. That is,

$$P_{v_i^j} = 1 - \frac{k_{v_i^j}}{\sum_{l=1}^{k_{v_i}} k_{v_i^l}} \tag{3}$$

  (iii) If the resulting network is not connected, recover the edge deleted in (ii) and go to step 1.

  (iv) Select a node that is not in $\Omega_{v_i}$, based on a probability proportional to its degree, to connect to $v_i$, and go to step 1.

If $\Omega^2_{v_i}/\Omega_{v_i}$ is empty, all $v_i$'s neighbors' neighbors are also $v_i$'s neighbors, and step 3(ii) is used to evaluate the number of shared members between $v_i$ and its nearest neighbors. In general, the above model evolves random networks using two kinds of interactions, in which the choice is governed by the only parameter, $P_{\text{local}}$. At each time step, only one edge is rewired. In local interactions, an edge is rewired to one of one end's neighbors' neighbors, while in global interactions, an edge is rewired to any node that is not one of the two ends' neighbors. However, in both interactions, the connectivity of the whole network is always maintained.

In local interactions, we used the analogy that we often become a friend's friend, while in global interactions, we used the idea of preferential attachments. Although each of these ideas has been studied in depth previously, our work focuses on studying their combined effect, since our model is completely characterized by the ratio of global to local interactions. In the following sections, we simulate the above model and study various properties of the generated networks.

## 3 Community Structure

A large body of work has been devoted to identifying community structures in networks. Newman and Girvan [41] introduced an elegant concept, the modularity $Q$, to measure the goodness of a community partition. Suppose a network is divided into $m$ disjoint communities. Then

$$Q = \sum_{i=1}^{m} \left( e_{ii} - a_i^2 \right) \tag{4}$$

where $e_{ii}$ is the fraction of edges that fall within community $i$, and $a_i$ is the fraction of all ends of edges that are attached to nodes in community $i$; that is, $a_i = \sum_j e_{ij}$.

Basically, $Q$ measures the deviation between the chance for edges among communities to be generated due to the community structure and the chance for the edges to be generated randomly. Extensive experimental results have demonstrated that $Q$ is an effective measure for community quality, in that a maximal $Q$ is a good indicator of the best community structure and therefore the best community number $m$. In practice, a value of $Q$ greater than 0.3 appears to indicate significant community structure, and the value of $Q$ for real-world networks typically falls in the range from 0.3 to 0.7. Higher values are rare.

In the following experiments, we use the method proposed in [40] to extract communities from networks at certain time steps, and $Q$ to evaluate the quality of the extracted communities. The first part is devoted to studying the community quality, and the second part focuses on community sizes.

### 3.1 Community Quality

Since $P_{\text{local}}$ is our sole parameter, we first study how $P_{\text{local}}$ affects community quality. We increase $P_{\text{local}}$ from 0.5 to 1.0 in steps of 0.05. The number of nodes and the average degree are set to 1000 and 10, respectively. For each different $P_{\text{local}}$ value, 10 graph instances are evolved independently, and to be sure that the structure of the random graph disappears, we run the model until $t_{\max} = 5 \times 10^4$. The obtained $Q$ values for each parameter setting are shown in Figure 1.

Figure 1 clearly shows the trend of $Q$ changing with $P_{\text{local}}$. The larger the value of $P_{\text{local}}$ is, the larger the value of $Q$ is. When $P_{\text{local}}$ is smaller than 0.75, $Q$ is always smaller than 0.3, which indicates no community structure emerging, while when $P_{\text{local}}$ is larger than 0.75, $Q$ is always larger than 0.3 and even increases to nearly 0.9 when $P_{\text{local}}$ reaches 1.00. That is to say, when $0.75 \leq P_{\text{local}} \leq 1.00$, significant community structures emerge in the evolving networks.

To get a further view on the dynamic of the evolutionary process of $Q$, Figure 2 shows how the values of $Q$ change with $t$ for different $P_{\text{local}}$ values. The $Q$ value is calculated every 1000 time steps and averaged over the 10 instances at the same time step. As can be seen, on starting from random networks, the evolution of $Q$ follows different paths depending on $P_{\text{local}}$. Since $t \geq 5 \times 10^4$, the value of $Q$ for $P_{\text{local}}$ such that $P_{\text{local}} \leq 0.70$ is always smaller than 0.3, while that for $P_{\text{local}} \geq 0.80$ is always larger than 0.3. The $Q$ for $P_{\text{local}} = 0.75$ fluctuates around 0.3. That is to say, when $P_{\text{local}} > 0.75$, the model starts to have the ability to reproduce a community structure. When we further look at the results in this parameter range, we find the evolution of $Q$ is also different. When $P_{\text{local}} = 0.80$, the evolution of $Q$ is quite stable, and is always in the range of 0.3–0.4. The cases when $P_{\text{local}} = 0.85$, 0.90, and 0.95 are similar. The value of $Q$ always first increases to a high value and then drops to the range of 0.3–0.5. High-quality communities are formed at the early stage of evolution, and then smaller communities merge to form larger communities, which results in a drop of $Q$. For $P_{\text{local}} = 1.00$, the value of $Q$ is always very high.

To show the scalability of the model, we increase the network size to 20,000 nodes. The evolutionary process of $Q$ is shown in Figure 3, and we can see that the results are similar. In addition,

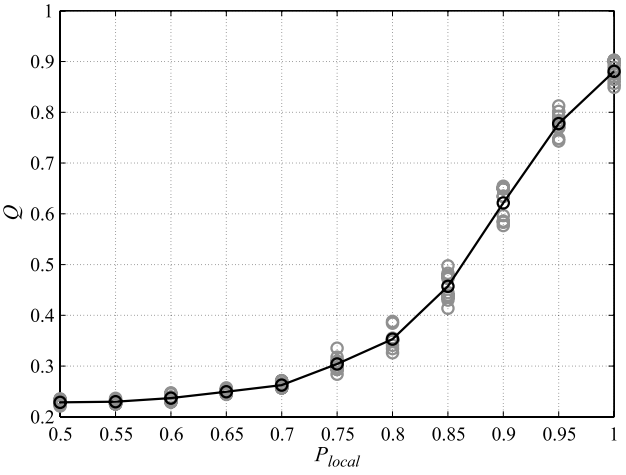


Figure 1. How the value of $Q$ changes with $P_{\text{local}}$, where $t_{\max} = 5 \times 10^4$, and gray circles represent the data for each of the 10 graph instances with lines connecting the means.

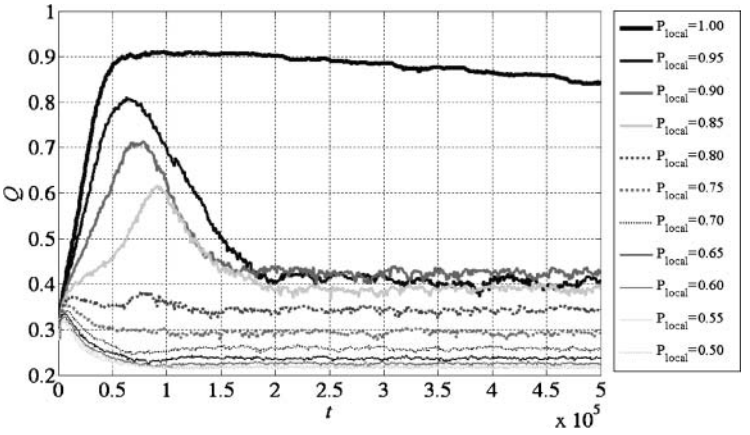


Figure 2. The evolution of $Q$ of networks with 1000 nodes.

we also calculated $Q$ of networks with the same edge density generated by the Barabási-Albert (BA) model [4], which are a typical type of scale-free networks. The results show that no matter what the number of nodes is (1000 or 20,000), $Q$ is always about 0.27; that is, no community structure emerged.

Similar to [24, 25, 55], the above results show that the community structure emerges naturally in our model, without the need of prelabeling a community for each node as in [58]. Clearly, the community formation in our model is based on the ratio of local to global interactions during the evolutionary process. When that ratio is above a threshold (0.75, or more safely 0.80), community structures emerge. Local interactions are responsible for the formation of community. However, if only local interactions are used, namely $P_{\text{local}} = 1.00$, the values of $Q$ are too high, which is rarely seen in real networks. Another indication of the above results is that as long as global interactions exist, small communities will merge to larger ones as the network evolves. Then, the community quality will be reduced, but still be above 0.3, which manifests a substantial community structure.

The above results accord with our knowledge about social interactions. Most of the time, people live in their own communities and contact those friends and colleagues whom they often meet; just

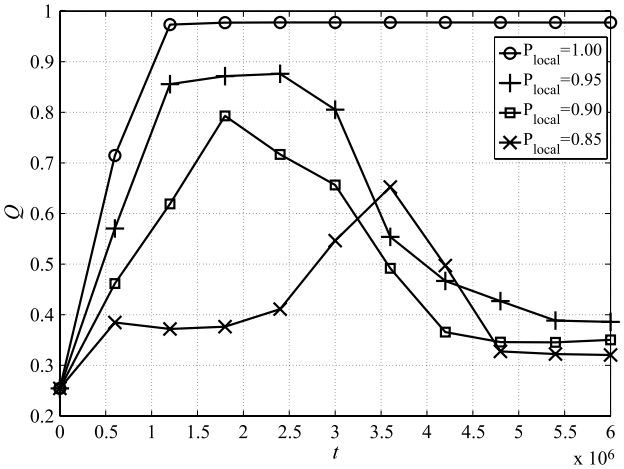


Figure 3. The evolution of $Q$ of networks with 20,000 nodes.

occasionally they attend meetings in other places. That is to say, the fraction of local interactions is much higher than that of global interactions. Furthermore, although the fraction of global interactions is low, it is an indispensable part of our life.

### 3.2 Community Size

We will check community size from two perspectives, namely the largest community size and the community size distribution. The above experimental results showed that global interactions are an indispensable part, since the community quality is too high to match the situation in real-world networks when only local interactions are used. Here, the following experiments will further show the role of the global interactions in forming communities. The evolution of the largest community size for different $P_{\text{local}}$ values is shown in Figure 4. The number of nodes is 1000, and the size is sampled every 1000 time steps and averaged over the 10 instances at the same time step.

Although the largest community size fluctuates with time, the general trend is still clear. We can see that the larger the value of $P_{\text{local}}$ is, the smaller the community size is. For $P_{\text{local}} = 1.00$, when $t > 10^5$, the community size is always smaller than 100. These results clearly show that global interactions are useful in forming large communities, which once again confirms the above conclusion that global interactions are indispensable.

Real networks are characterized by heterogeneous distributions of node degree. Likewise, it is not correct to assume that all communities have the same size. In fact, the distribution of community sizes of real networks is also broad, with a tail that can be fairly well approximated by a power law [10, 11, 26, 45]. Figure 5 shows the community size distribution in networks with 20,000 nodes for $P_{\text{local}}$ such that $0.85 \leq P_{\text{local}} \leq 1.00$ when $t = 3 \times 10^7$.

As can be seen, when $P_{\text{local}} = 0.85$, 0.90, and 0.95, the community size distributions can be approximated by power law distributions with exponents 3.49, 2.92, and 3.08, respectively; that is, most communities are small in size, while only a few large communities exist, which matches real-world situations. When $P_{\text{local}} = 1.00$, the distribution can still be approximated by a power law distribution without taking into account the leftmost data point.

## 4 Degree Distribution

Figure 6 shows the degree distribution of networks with 20,000 nodes when $P_{\text{local}} = 0.85$, 0.90, 0.95, and 1.00, where $t = 3 \times 10^7$. As can be seen, when $P_{\text{local}} = 0.85$, 0.90, and 0.95, the degree

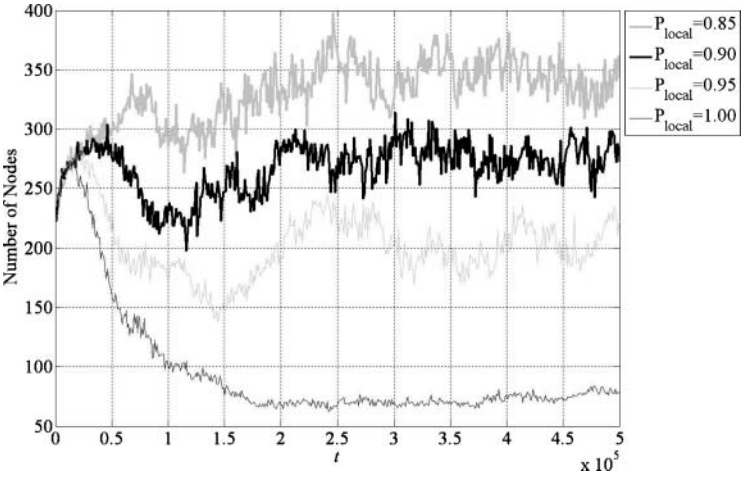


Figure 4. The evolution of the largest community size in networks with 1000 nodes.

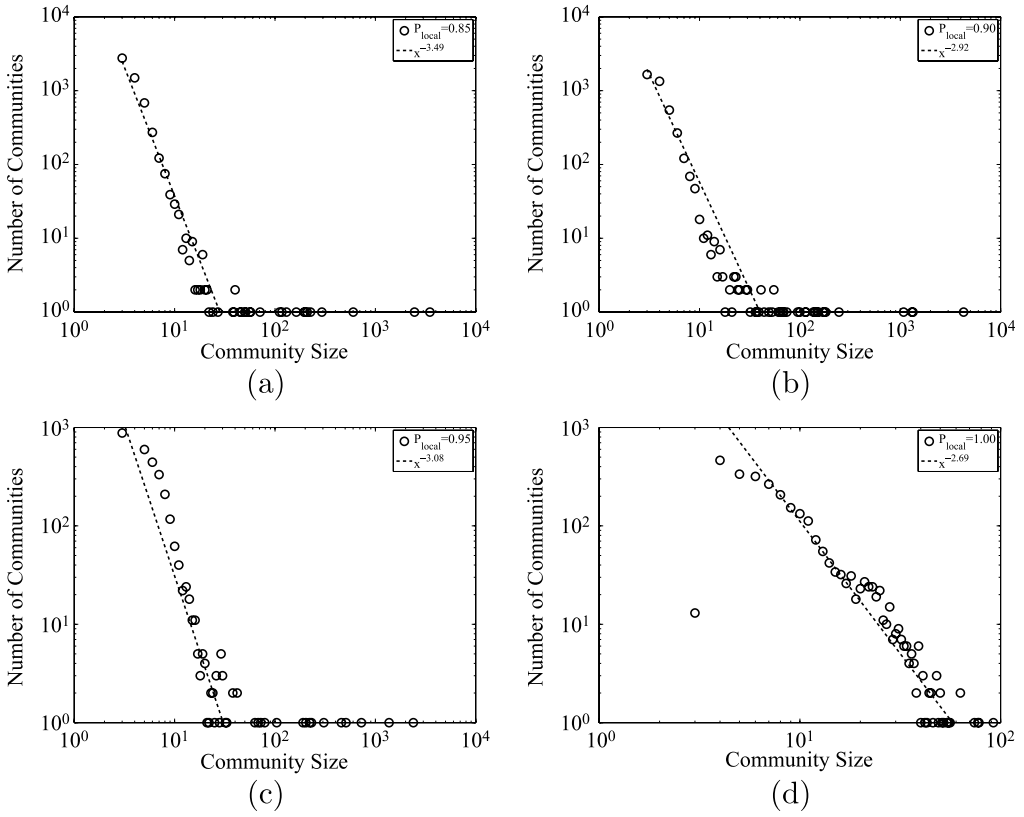


Figure 5. The community size distribution of networks with 20,000 nodes when $t = 3 \times 10^7$ for $P_{\text{local}}$ = (a) 0.85, (b) 0.90, (c) 0.95, and (d) 1.00.

distribution can be approximated by power law distributions with exponents 1.94, 1.77, and 1.95, respectively. But when $P_{\text{local}} = 1.00$, the model fails to form an approximate power law distribution. These results confirm the roles of global and local interactions we found in previous experiments. Although local interactions are useful in creating the community structure, without global interactions, local interactions cannot create a network with both community structure and power law degree distribution.

## 5 Assortative Mixing

Another important network feature is the correlation between properties of adjacent network nodes, which is known in the ecology and epidemiology literature as *assortative mixing*. Assortative mixing can have a profound effect on the structural properties of a network. For example, assortative mixing of a network by a discrete characteristic will tend to break the network up into separate communities [38]. Newman [37] defines the assortativity coefficient $r$ of an undirected network as the Pearson correlation coefficient of the degrees at the ends of an edge:

$$r = \frac{M^{-1}\sum_{i=2}^{N}\sum_{j=1}^{i} a_{v_i v_j} k_{v_i} k_{v_j} - \left[M^{-1}\sum_{i=2}^{N}\sum_{j=1}^{i} \frac{1}{2} a_{v_i v_j}\left(k_{v_i} + k_{v_j}\right)\right]^2}{M^{-1}\sum_{i=2}^{N}\sum_{j=1}^{i} \frac{1}{2} a_{v_i v_j}\left(k_{v_i}^2 + k_{v_j}^2\right) - \left[M^{-1}\sum_{i=2}^{N}\sum_{j=1}^{i} \frac{1}{2} a_{v_i v_j}\left(k_{v_i} + k_{v_j}\right)\right]^2} \tag{5}$$

where $a_{v_i v_j} = 1$ when the edge between $v_i$ and $v_j$ exists, and $a_{v_i v_j} = 0$ otherwise. $r$ has a value in the range $[-1, 1]$, with positive assortativity indicating that nodes preferentially link to nodes with similar degrees.

The evolution of $r$ in networks with 1000 nodes for $P_{\text{local}}$ such that $0.85 \leq P_{\text{local}} \leq 1.00$ is shown in Figure 7, which is sampled every 1000 time steps and averaged over 10 instances at the same time step. All values of $r$ are always larger than 0 when $t \geq 5 \times 10^4$, which indicates that nodes are preferentially linked to nodes with similar degree. Moreover, the relationship between local interactions and the assortativity coefficient is clearly shown; namely, the larger the amount of local interactions is, the higher the value of $r$ is.

Recently, Piraveenan et al. [48] argued that the above network-level measure conveys insufficient information about the local level structure and motifs present in networks. They introduced a measure of *local assortativeness* that quantifies the level of assortative mixing for individual nodes in the context of the overall network, and used this measure to study the growth of the Internet [49]. Since two networks with the same network assortativeness and similar degree distributions may have entirely different local assortativeness distributions [48], next, we further study local assortativeness of networks generated by our model.

Local assortativeness of a given node $v$ is defined as follows [50]:

$$\rho_v = \frac{j(j+1)\left(\bar{k} - \mu_q\right)}{2M\sigma_q^2} \tag{6}$$

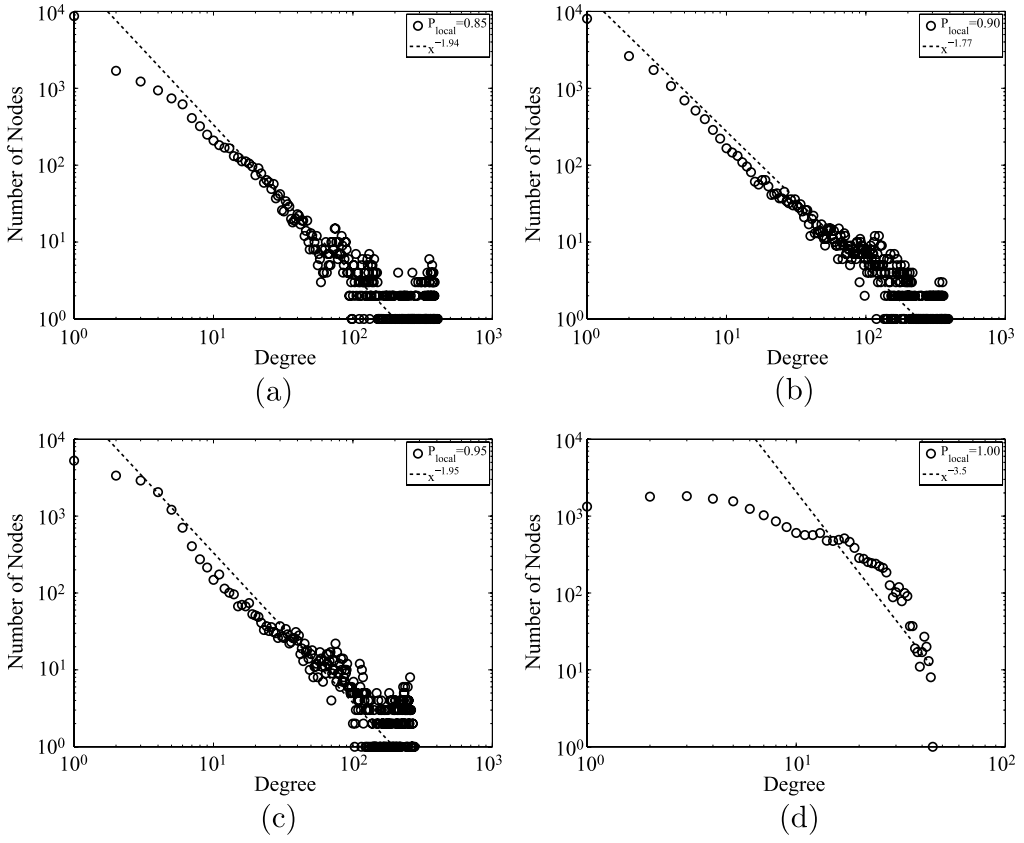


Figure 6. The degree distribution of networks with 20,000 nodes when $t = 3 \times 10^7$ for $P_{\text{local}} =$ (a) 0.85, (b) 0.90, (c) 0.95, and (d) 1.00.

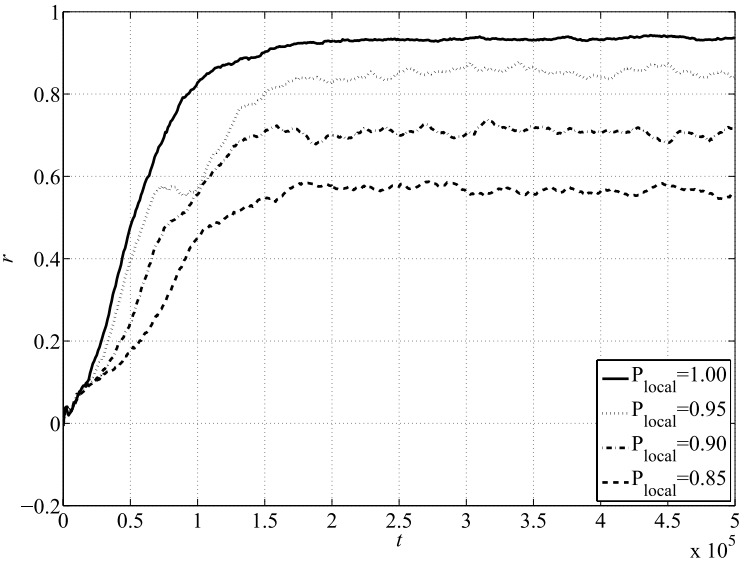


Figure 7. The evolution of the assortativity coefficient in networks with 1000 nodes.

where $j$ is the node's excess degree, $\bar{k}$ is the average excess degree of its neighbors, $\mu_q$ is the expectation of the excess degree distribution, $\sigma_q$ is the standard deviation of this distribution, and $M$ is the number of edges in the network. In fact, in a network with $N$ nodes, we have

$$r = \sum_{i=1}^{N} \rho_i \tag{7}$$

Since local assortativeness is a property of a node, it is possible to construct local assortativeness distributions for a given network, enabling us to plot local assortativeness values against degrees. Figure 8 shows the local assortativeness distributions of networks with 20,000 nodes, where the values are averaged over all nodes with the same degree.

As can be seen, in networks generated by our model, the local assortativeness increases with degree in general. Moreover, the larger the value of $P_{\text{local}}$ is, the clearer this pattern is. We also show the result of the BA model. Since the assortativity coefficient of the BA model is nearly 0, there is no clear relationship between local assortativeness and degree. Furthermore, in Figure 8b, we zoom in on the part corresponding to degrees from 0 to 200. We can see that although the assortativity coefficient of the network generated by our model is positive, a large number of disassortative nodes ($\rho < 0$) exists, no matter what the value of $P_{\text{local}}$ is, which confirms the observation in [48].

## 6 Internet Topology and the Rich-Club Phenomenon

In this section, networks generated by our model are compared with real-world Internet topology at the autonomous system (AS) level, namely, the AS graph. In this graph, each node represents an autonomous system present in the Internet, and the edges represent a commercial agreement between two Internet service providers (which own the two ASs). Such an agreement specifies whether they agree to exchange data traffic and how to charge each other. Thus the AS graph is the "control plane" of the Internet and presents very realistic opportunities to gain insight into the evolution of complex networks. The growth of this network has been well documented, snapshots

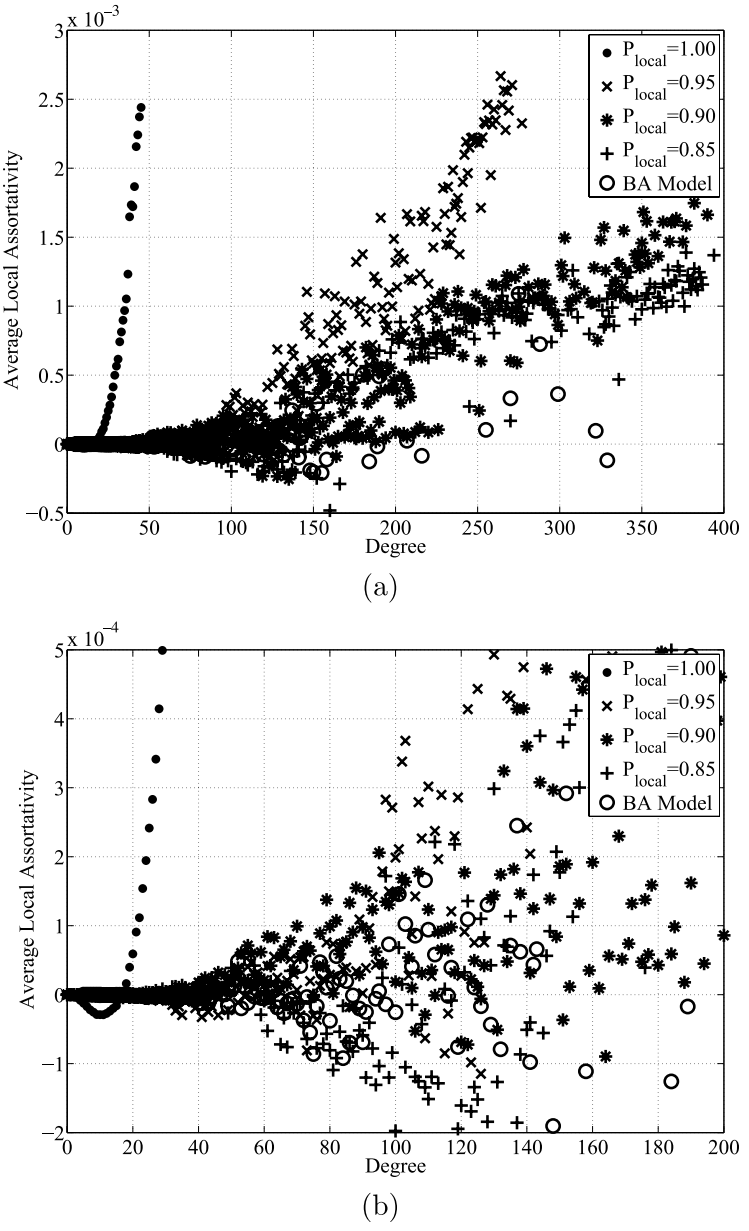


Figure 8. The local assortativeness distributions of networks with 20,000 nodes when $t = 3 \times 10^7$, where (b) is a magnified part of (a).

of the network being available on a regular basis [1]. The AS graph used in the following studies is the one used in [63], which has 11,461 nodes and 32,730 edges, and our model and the BA model use the same numbers of nodes and edges.

First, Table 1 compares the community structure in the AS graph, the BA model, and our model. The results show that the value of $Q$ in the AS graph is equal to 0.5719, which is closer to the cases when $P_{\text{local}} = 0.85$ and 0.90 (0.6296 and 0.5339, respectively). When $P_{\text{local}} = 0.95$ and 1.00, $Q$ is much higher than that in the AS graph, while $Q$ in the BA model (0.3862) is much lower. Next, Figure 9 compares the degree distributions in the AS graph, the BA model, and our model. As can

Table 1. The community structure in the AS graph, our model ($t = 11{,}461 \times 80$), and the BA model.

| Network | | $Q$ |
|---|---|---|
| AS graph | | 0.5719 |
| Our model: | $P_{local}$ = 0.85 | 0.6296 |
| | 0.90 | 0.5339 |
| | 0.95 | 0.8983 |
| | 1.00 | 0.9786 |
| BA model | | 0.3862 |

be seen, when $P_{local} = 0.85$ and 0.90, the degree distribution in our model is closer to that in the AS graph.

Another interesting property of the AS graph is the rich-club phenomenon; that is, the rich nodes, which are a small number of nodes with large numbers of links, are very well connected to each other [63]. The rich-club phenomenon is a simple qualitative way to differentiate between power law topologies and provides a criterion for new network models. Therefore, Figure 10 checks whether this phenomenon exists in our model, where the rich-club connectivity is calculated according to [63]. That is to say, nodes in the network are sorted by decreasing number of links that each node contains. The rich club is defined as nodes with rank less than $r_{max}$, the normalized position of a node on the ordered list. The rich-club connectivity is defined as the ratio of the total actual number of links to the maximum possible number of links between members of the rich club. As can be seen, our model also reproduces the rich-club phenomenon. Moreover, when $P_{local} = 0.85$, 0.90, and 0.95, the rich-club connectivity curves are much closer to that in the AS graph.

Finally, Table 2 further compares the properties of the AS graph with those of networks generated by different models, where the data of Inet-3.0 [60] and the fitness BA model [8] are cited from [63], $l(r_i \leq 5\%, r_j)$ is the number of links connecting with the top 5% rich nodes, and $l(r_i \leq 5\%, r_j \leq 5\%)$ is the number of links connecting between the top 5% rich nodes. The results show that the values of $l(r_i \leq 5\%, r_j)$ in the networks generated by our model with $P_{local} = 0.85$ and 0.90 are as close as that in Inet-3.0, which are closest to that in the AS graph, while the values when $P_{local} = 0.95$ and 1.00 are much smaller. The values of $l(r_i \leq 5\%, r_j \leq 5\%)$ when $P_{local} = 0.85$, 0.90, and 0.95 are larger than 10,000, but are closer to the value of the AS graph (8910) than that of Inet-3.0, which is only 3697. In terms of the maximum node degree, our model is outperformed by Inet-3.0.

To summarize, all the above comparisons between our model and the AS graph show that our model with $0.85 \leq P_{local} \leq 0.95$ can generate networks that reproduce many properties observed in AS graph, except the maximum node degree. This further confirms the roles of global and local interactions.

## 7 Discussion and Conclusion

In this article, a network rewiring model is proposed, based on a simple view of social interactions. Extensive simulation results show that by varying only the parameter $P_{local}$, the model can evolve from random networks to those with structural characteristics closely matching empirical social networks. The results show that when $P_{local}$ is larger than 0.85, significant community structure always

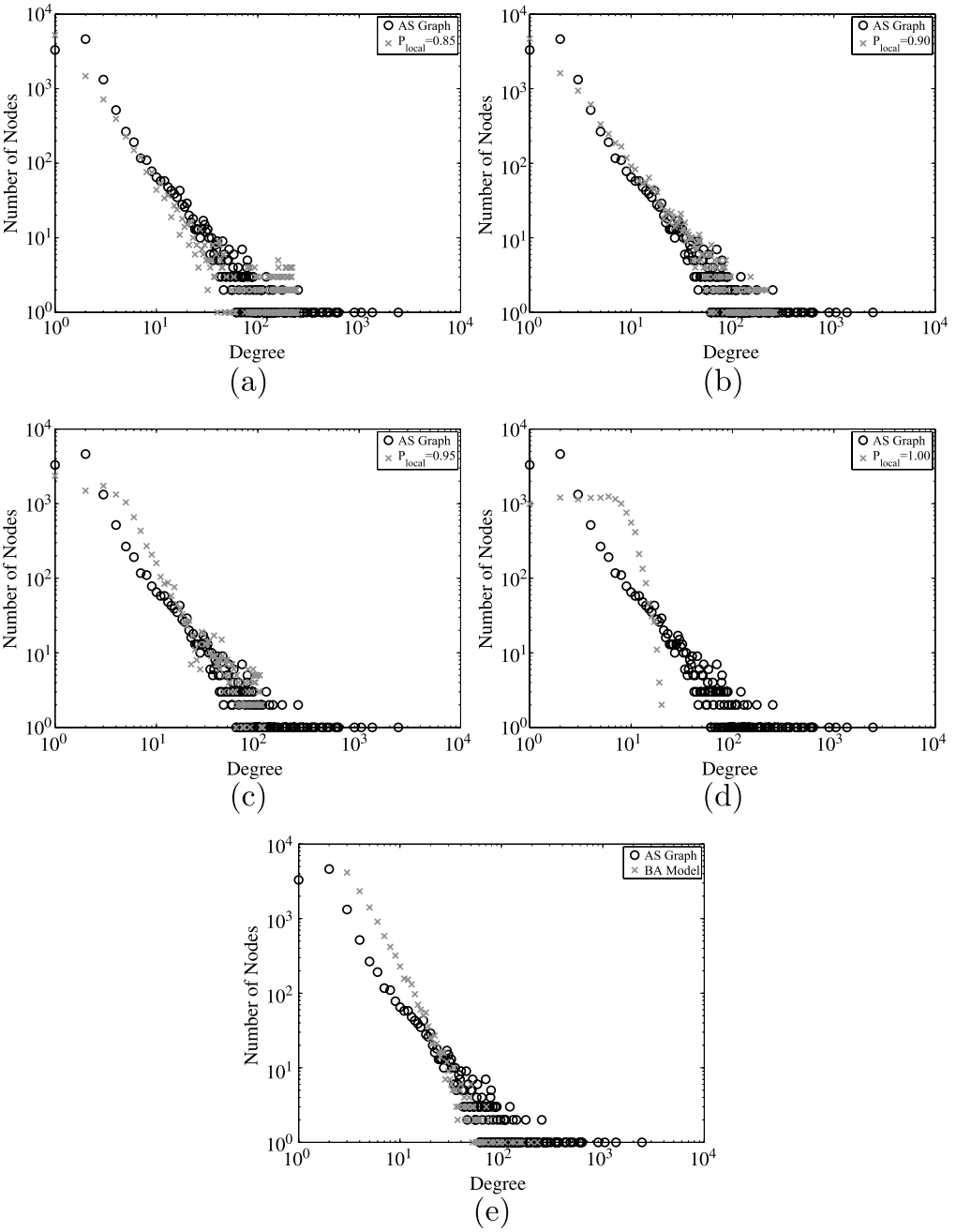


Figure 9. The degree distributions in the AS graph, the BA model, and our model ($t = 11461 \times 80$). (a) $P_{local} = 0.85$. (b) $P_{local} = 0.90$. (c) $P_{local} = 0.95$. (d) $P_{local} = 1.00$. (e) The BA model.

emerges, and as long as $P_{local}$ is smaller than 0.95, both the community size distribution and the degree distribution can be approximated by a power law distribution. In the above range of $P_{local}$, the model also exhibits positive assortative mixing by degree and the rich-club phenomenon.

The results clearly show the different roles of local and global interactions. Local interactions are responsible for the formation of the community structure, and have a positive correlation with assortative mixing. Meanwhile, global interactions play an important role in forming large communities

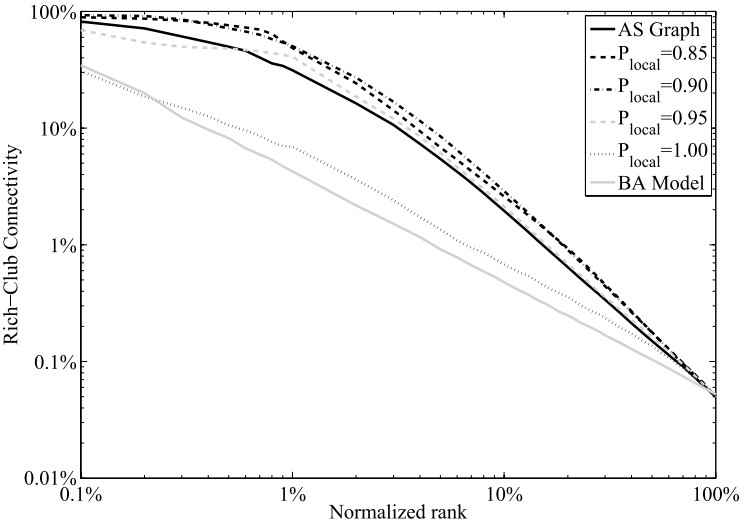


Figure 10. Rich-club connectivity against node rank.

Table 2. Comparison between properties of the AS graph and other models.

| Network | $M$ | $I(r_i \leq 5\%, r_j)$ | $I(r_i \leq 5\%, r_j \leq 5\%)$ | Maximum node degree | Average node degree |
|---|---|---|---|---|---|
| AS Graph | 32,730 | 28,610 | 8,910 | 2,432 | 5.7 |
| Our model: $P_{local}$ = 0.85 | 32,730 | 22,725 | 11,189 | 198 | 5.7 |
| 0.90 | 32,730 | 22,664 | 13,935 | 267 | 5.7 |
| 0.95 | 32,730 | 15,249 | 10,009 | 112 | 5.7 |
| 1.00 | 32,730 | 5,446 | 2,216 | 20 | 5.7 |
| BA model | 34,377 | 13,710 | 1,498 | 227 | 6.0 |
| Inet-3.0 | 24,171 | 22,620 | 3,697 | 2,010 | 4.2 |
| Fitness BA | 34,366 | 20,929 | 1,426 | 1,793 | 6.0 |

and evolving the community size distribution and the degree distribution to a power law distribution. In general, both local and global interactions are indispensable in evolving the model from random networks to those matching various properties exhibited by the real-world networks, and the fraction of local interactions should be much higher than that of global interactions. These results accord with our experience in the real world. That is, we make local contacts, and just occasionally we contact someone far away from us.

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