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The Final Ritard: On Music, Motion, and Kinematic Models

Motion plays an important role in music, a fact evidenced not only by the wealth of terminology used by musicians and music theorists that refer to music in ''motional'' terms. Consider, for example, how we speak of music as ''slowing down,'' ''speeding up,'' ''moving from F-sharp to G,'' etc. A considerable amount of theoretical and empirical work tries to illustrate apparent relation between physical motion and music (see Shove and Repp 1995 for an overview). However, it is very difficult to specify—let alone validate—the nature of this long-assumed relationship. Is there a true perceptual experience of movement when listening to music, or is it merely a metaphorical one owing to associations with physical or human motion?

Some scientists have looked at music and motion in a very direct way, for instance, relating walking speed to preferred tempi (e.g., Van Noorden and Moelants 1999) or body size to timing patterns found in music (Todd 1999). However, these direct relationships between the human body and music seem too simplistic to generally hold. Others have approached the relation more as a metaphorical one, arguing that musicians allude to physical motion in their performances, imitating it in a musical way (cf. Shove and Repp 1995). These theories tend to be difficult to express in computational terms.

This article reviews a family of computational models (e.g., Sundberg and Verillo 1980; Feldman, Epstein, and Richards 1992; Todd 1992; Friberg and Sundberg 1999) that do make the relation between motion and music explicit and therefore can be tested and validated on real performance data. These kinematic models attempt to predict the timing patterns found in musical performances (generally referred to as *expressive timing*). Most of these studies focus on modeling the *final ritard*: the typical slowing down at the end of a music performance, especially in music from the Western Baroque and Romantic periods. But this characteristic slowing down can also be found in, for instance, Javanese gamelan music or some pop and jazz genres. In this kinematic approach, one looks for an explanation in terms of the rules of mechanics: that is, how expressive timing might relate to, or can be explained by, models of physical motion that deal with force, mass, and movement.

A discussion of these kinematic models is presented below in the form of a story (see Figure 1), with three fictitious characters who represent the different disciplines involved in this research (psychology, mathematics, and musicology). The story is a continuation of Desain and Honing (1993; see also http://www.nici.kun.nl/mmm/tc for additional sound examples), an article that dealt with the state of the art in expressive timing research some ten years ago. In addition, it brought forward a critique on the usefulness of the *tempo curve* (a continuous function of time or score position) as the underlying representation of several computational models (including most computer music software at that period). The main point of critique was that the predictions made by models using this representation are insensitive to the actual rhythmic structure of the musical material: they make the same predictions for different rhythms. All this suggested the existence of a richer representation of timing in music perception and performance than is captured by an unstructured tempo curve.

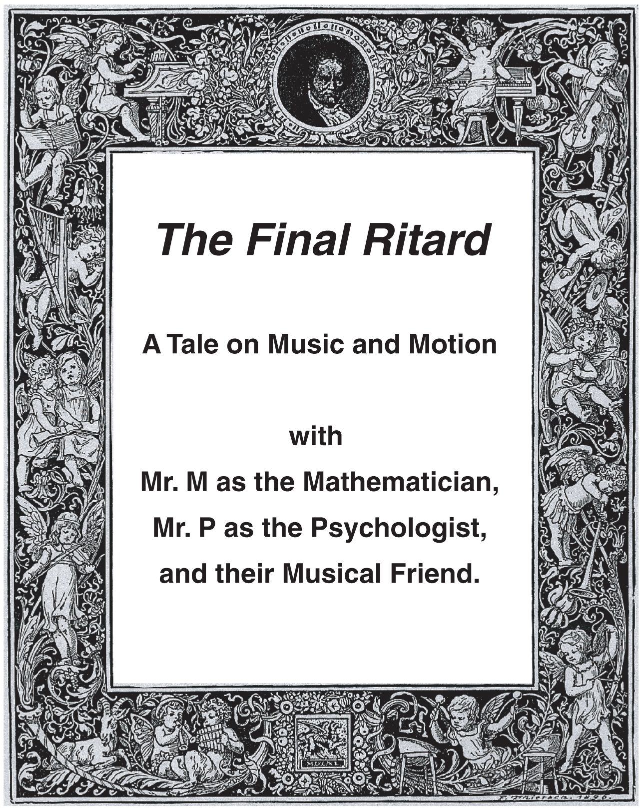
The present article attempts to offer an informative but informal discussion of models of the final ritard, including some of the problems that these kinematic models do not address. Experimental support for an alternative view, as briefly presented in the discussion, will be the topic of a forthcoming article.

The Final Ritard: A Tale on Music and Motion

In the following text, P, M, and their musical friend MF continue their enthusiastic search in trying to

Computer Music Journal, 27:3, pp. 66–72, Fall 2003

Figure 1. The final ritard, a tale on music and motion.



unravel the mystery of timing in music performance. This time they will find out about the kinematic approach to expressive timing and computational models that are also based on the notion of a tempo curve; as such they are likely to continue their argument.

Prologue: What Happened Before

Quite some time ago, P, who is interested in psychology, and M, an amateur mathematician, got together during the Christmas holidays with their musical friend MF. Those were the days before cellular telephones, a time of herbal tea and the just-arrived technology of MIDI. MF, while duly impressed by P's and M's well-equipped music studio and expertise in computer modeling, remained unimpressed by their musical results and, sadly, left, rather irritated, to spend his Christmas elsewhere.

Part 1: In Which MF Had an Important Insight and P Found the Appropriate Literature

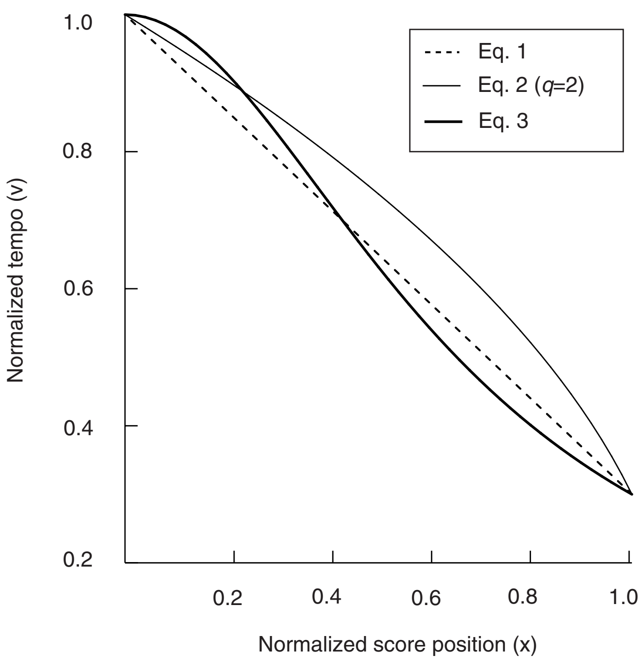
Not so long ago, MF remembered those Christmas holidays while he was reading a book on the history of tempo rubato. He was still convinced his friends were on the wrong track with their silly computer models. But the more he read about tempo rubato, the more he was convinced that they might have overlooked an obvious link between music and biological motion. Blatantly obvious (once he realized it) was the explicit reference of much music terminology—words like andante or accelerando—to qualities of human movement. And therefore, he reasoned, a successful model of expressive timing—unlike the unsuccessful models made by his friends—should be based on the rules of movement and the human body.

MF couldn't help making a phone call to P, the amateur psychologist, to tell him about his new insight. "My dear friend P," he said, "for expressive timing to sound natural in a performance, it must conform to the principles of human movement. Isn't knowledge about the body—the way it feels, moves, reacts—what musicians share with their listeners?" P almost immediately became enthusiastic. He saw a new opportunity to continue the investigations that had ended so brusquely before. P decided to go to the library, and there he found a lot of interesting psychological literature on the relation between motion and music. Much of it, however, involved some formidable mathematics. MF then proposed to have a new gathering with the "old team," including their mathematical friend, this time at MF's home, safe from modern technology!

Part 2: In Which the Friends Met Again and Explored Elementary Mechanics

A few days later, P and M found themselves at MF's kitchen table, which was well stocked with a pot of tea and a tin full of cookies. They returned to a lively discussion of expressive timing in music. After browsing through the books that P brought, M (the amateur mathematician) stated

Figure 2. Prediction of the final ritard by the kinematic models described in Equations 1, 2, and 3 (with w = *0.3). Tempo and score position are normalized.*



with some authority, "These models borrow from elementary mechanics and kinematics. They talk about mass, force, and speed of an object in terms of velocity, time, and place. And, interestingly, tempo variations in music performance are compared with the behavior of physical objects in the real world." P was all ears; MF just took another sip of his tea.

M wrote most of the formulas, one below the other, on a piece of paper, patiently explaining their formal differences. A tidier version of M's jottings is given next.

Interlude: Formalizations of the Final Ritard

Now, some of the existing formalizations of the final ritard are briefly summarized. Kronman and Sundberg (1987) define the final ritard as a square root of score position, a model of constant braking force (a convex function; see Figure 2):

$$v(x) = (u^2 + 2ax)^{1/2} \tag{1a}$$

where v is velocity (or tempo), x is distance (or score position), u is initial tempo, and a is acceleration.

Longuet-Higgins and Lisle (1989) and Todd (1992) propose an identical model, but express it rather as tempo (v) linear in time (t):

$$v(t) = u + at \tag{1b}$$

Friberg and Sundberg (1999) generalize this model by adding a variable q for curvature (varying from linear to convex shapes; see Figure 2), w (a non-zero final tempo), and normalize it:

$$v(x) = [1 + (w^q - 1)x]^{1/q} \tag{2}$$

Todd (1985) and Repp (1992) suggest quadratic Inter-Onset Interval (IOI, or beat duration) as a function of score position:

$$IOI(x) = c + kx + lx^2 \tag{3}$$

where c is a constant reflecting vertical displacement, and k and l are coefficients reflecting the degree of curvature. This results in a concave function when expressed as tempo as a function of score position (see Figure 2). In addition, Feldman, Epstein, and Richards (1992) and Epstein (1994) discuss a model of force dynamics. However, they tested it with a model of beat duration that is in fact unrelated to a model of force, just like Equation 3 (cf. Friberg and Sundberg 1999). Figure 2 illustrates the equations above.

Part 3: In Which the Friends Built a "True" Physical Model

After seeing so many formulas and equations, MF protested "But M, please! We are investigating music here, not mechanics!" "Look," P swiftly interrupted, "I found the studies of these music researchers. They explain ritardandi in music performance as alluding to human motion, like the way runners come to a standstill. Let me read a passage for you: 'Performers aim at this allusion, and listeners, with some education, find it aesthetically pleasing' (Repp 1992). Isn't this exactly what you described to me on the phone?"

P and M seemed confident that they had now found what they had been searching for all the time. MF too was quite pleased with the fact that

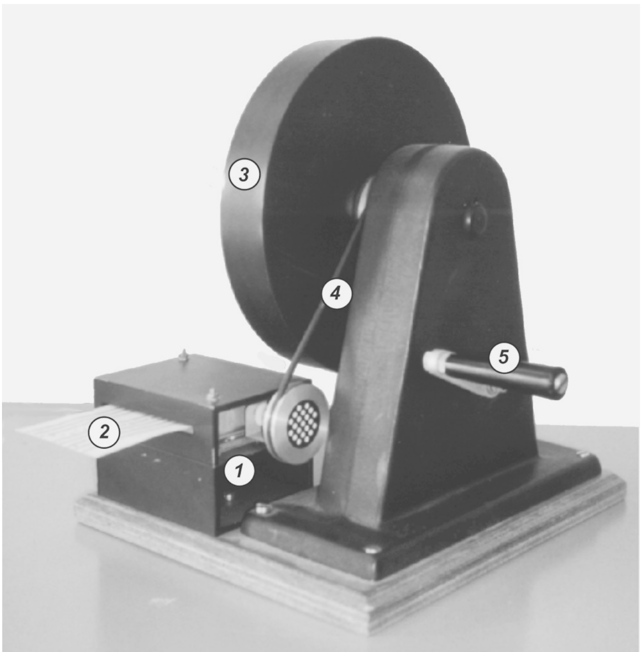


Figure 3. A mechanical implementation of a constant braking force model, consisting of a music box (1), a piece of piano roll (2), solid-metal flywheel (3), belt (4), and a handle (5). For a short movie showing the machine at work, see www.hum.uva.nl/mmm/fr/.

these respected researchers had found evidence for his intuitive ideas about bodily motion. But he still had reservations. ''How does the math of elementary mechanics compare to a final ritard in music? Can't we listen to these formulas?'' M replied with a frown on his face, ''Well, if we would have met in our studio, we could have programmed them for you. Now, we must think of something else.'' But after a small pause he began to smile. ''Let's see how far we can get with the material in your garage.''

That morning, MF's kitchen turned into a real workshop. ''Can we use one of your music boxes?'' P asked sheepishly. With some hesitation, MF collected one of his beloved machines from the living room. And after some hours of trifling and hammering, they had built it—a ''true'' physical model of constant braking force! (See Figure 3.)

The machine they built contained a music box with the crank replaced by a flywheel. This flywheel was connected to the music box with a belt, as shown in Figure 3. When turning the new handle, the music box would start playing, and when released—owing to the inertia of the flywheel—it would continue playing, slowly coming to a halt from the friction of the machinery.

MF inserted his favorite piano roll, a Bach fugue, into their newly made contraption. He turned the flywheel, and the music started playing. A few bars before the end, he released the handle, and the music came slowly to a standstill over the last few notes. ''Wonderful, wonderful!'' They all jumped with joy. MF thought his antique music box had finally become truly musical.

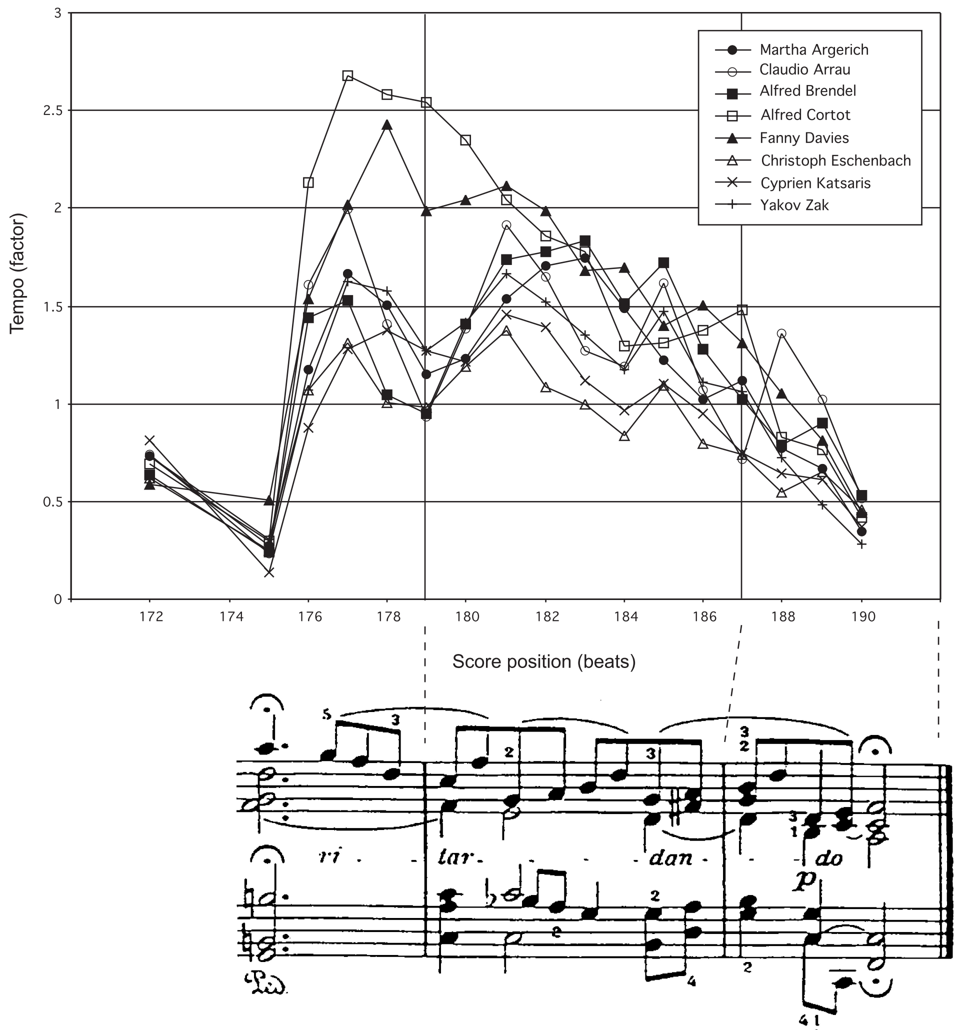
Part 4: In Which Some Disappointment Was Unavoidable and They Decided to Look at Real Performances

When they had calmed down a bit, M had a second look at his paper full of formulas, and said with a tone not atypical of a young mathematician, ''But I have to say that these models are actually underspecified. They make no claims about how to derive the 'metaphorical' mass or speed from the music. In our contraption, we just arbitrarily decided on the mass of flywheel, and we can freely decide the speed at which the handle is released.'' M also realized that their contraption had some shortcomings. ''Our flywheel has a fixed braking force, caused by the friction of the contraption. But it should actually be dependent on when and at what speed you release the handle and stop when the right final tempo is reached, like the equations show. That's difficult to make mechanically.''

But P responded ''Oh come on M, don't be so strict. Let's just try another one, a slightly more modern piece. What do you think?'' After some searching, MF returned with a piano roll of Beethoven's *Paisiello Variations*. ''Remember this?'' he teased, alluding to their previous Christmastime investigations using the same piece. MF inserted the piano roll, and they listened again for the last measures of each variation. But whatever they tried, releasing the handle early or late, at higher or lower speeds, it never sounded quite right. ''It doesn't do the rhythmic figures right,'' MF complained. ''Apparently, it only works with the repeated eighth notes of the fugue.''

''We could be here forever trying to change this or that factor,'' P warned. He was convinced they

Figure 4. Final ritards in performances of the last three measures of Schumann's Träumerei *from* Kinderszenen, *Op.15. (Tempo 1 is M.M. = 60; after Repp 1992.)*



had to return to the empirical approach. ''Why don't we look at how MF performs final ritards?''

Part 5: In Which They Looked at Graphs from Famous Pianists, But Couldn't Please Their Musical Friend

P opened his briefcase and removed a folder with the performance data they had collected during that first Christmas gathering. ''These are the graphs of MF performing the final measures of *Träumerei* by Schumann.'' And enthusiastically holding up an article, P added, ''And here are some interesting measurements made from recordings by some of your colleagues. Look, you played it just like Alfred Brendel!'' (See Figure 4.)

There was quite some diversity among these famous pianists; they all seemed to play the final measures differently. MF said questioningly, ''I do not see how one single curve could describe all these performances.'' P responded, ''But the point here is to model the average, normative performance,'' to which M added, while pointing at Equation 3, ''This research showed that the last six notes of these averaged performances can be fitted closely by a quadratic function. That is an important finding, isn't it?'' ''Indeed, M,'' P confirmed, ''but we must be aware that an average curve is a statistical abstraction, not a musical reality.''

Their musical friend smiled and took another close look at the diagrams. ''So if I understood your explanations,'' he asked M, ''this function should have a hollow, concave shape. But doesn't our contraption generate a convex-shaped deceleration?'' M confirmed this. ''A convex shape indeed is what the other research found. Apparently, there is evidence for a variety of shapes. However, what worries me is the complete freedom in deciding on the mass and amount of force applied; fitting these curves to the data is too flexible.'' ''Maybe all these pianists have their own specific force and mass,'' MF interjected optimistically. They looked at each other with some disappointment. It seemed that once again they had failed to find a model of expressive timing that could please their musical friend. MF, who this time wanted to end their endeavors in a more optimistic manner, proposed ''Let's go to the living room. I will play my favorite fugue for you.''

Discussion

This tale addresses kinematical models of expressive timing, and it questions how well expressive timing can be explained by models of physical motion. One point of critique is that the predictions made by these models are insensitive to the actual rhythmic structure of the musical material. This was stated more generally with respect to tempo curves in the original article (Desain and Honing 1993) and elaborated upon subsequently (Desain and Honing 1994; Honing 2001). However, more central is the concern that these descriptions do not, in principle, teach us anything about the nature (whether ''motional'' or not) of the underlying perceptual or cognitive mechanisms. Even if we assume that these tempo curves do give a good approximation of the empirical data (despite the contrasting results in the research discussed above), the mere fact that the overall shape (e.g., a square-root function) can be predicted by the rules that come with human motion is not enough evidence for an underlying physical model of expressive timing, however attractive such a model might be.

An alternative explanation could be based on the relation between rhythmic structure and expressive timing (Desain and Honing 1996). For example, a ritard of many eighth notes can have a deep rubato, while one of only a few notes and possibly a more elaborated rhythmical structure (i.e., with differentiated durations), might be less deep (i.e., exhibit less ''slowing down'' and/or ''speeding up''). Along these lines, it is not a class of functions (originating from mechanics) that best describes the timing patterns observed but a set of constraints that describe the boundaries of possible final ritards: the constraints on expressive timing are a consequence of the need not to break the perceptual rhythmic categories while decelerating quickly. (For example, slowing down more would be perceived as a different rhythm altogether.)

Models of tempo tracking and rhythmic categorization (e.g., Longuet-Higgins 1987; Desain and Honing 2001) predict the boundaries for which the rhythmical structure can still be perceived. Apart from explaining the dependency of a ritard on the performed rhythmic material, this yields constraints on the shape of the ritard. Such restrictions are not made by a physical motion model, because any metaphorical mass, force, and amount of deceleration are equally likely. As such, a final ritard might coarsely resemble a square-root function, with the added characteristic that the detail depends on the rhythmical material in question.

Finally, this does not mean that all timing patterns in music performance can be solely explained in terms of musical structure alone; therefore, the role of the body (Clarke 1993), its physical properties (Todd 1999), and the way it interacts with a musical instrument (Baily 1985) is too evident. The challenge is to construct a theory of music cognition that incorporates both the cognitive and embodied aspects of music perception and performance.

Acknowledgments

Special thanks to Robert Gjerdingen and Doug Keislar for valuable suggestions on an earlier version of this article, and to Bruno Repp for his constructive criticisms and for kindly providing the original data for Figure 4. We also thank the Department of Mechanics, University of Amsterdam, for actually making the contraption shown in Figure 3. And last but not least, thanks to Peter Desain with whom the characters of P, M, and MF were invented.

This article is based on a text first published in 2003 in *Music Theory Online* 9(1), (available online at societymusictheory.org/mto/issues/mto.03.9.1/toc.9.1.html). It was written during a sabbatical at New York University by kind invitation of Robert Rowe. The research was funded by the Netherlands Organization for Scientific Research (NWO) in the context of the Music, Mind, Machine project.

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